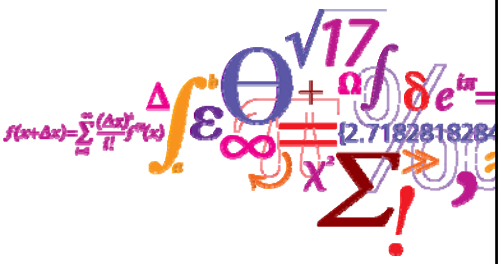
HAWCStab2 – A high-order linear wind turbine model for aeroservoelastic stability analysis and control design

Morten Hartvig Hansen



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Outline

- What is HAWCStab2?
  - model assumptions
  - open-loop equations
  - closed-loop equations
  - validation against HAWC2
- Turbine modal dynamics
  - terminology
  - structural modes
  - open- and closed-loop modes
- Examples
  - swept blades
  - flutter analysis
  - controller tuning
- Exercise
  - input for HAWCStab2
  - using it, e.g. flutter analysis

Closed-loop tower mode

What is HAWCStab2?

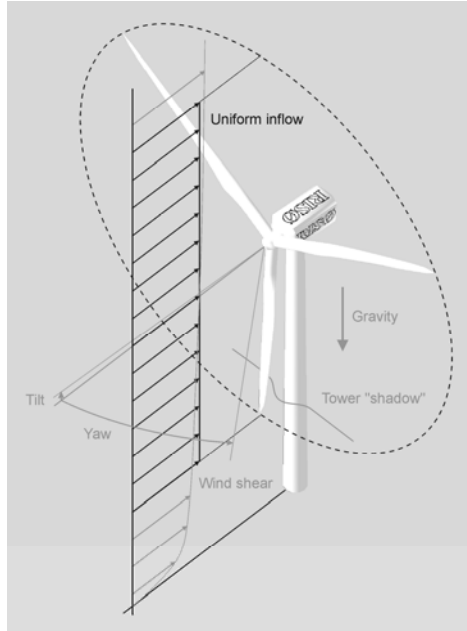
Linear aeroelastic model for eigenvalue and frequency-domain analysis of wind turbines

Kinematics based on a nonlinear co-rotational finite beam element formulation.

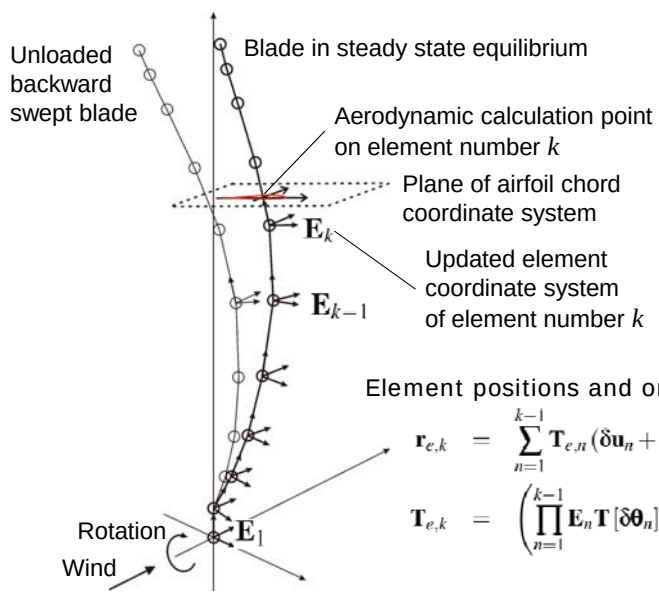
Uniform inflow to give a stationary steady state.

Analytical linearization about the stationary steady state.

Unsteady aerodynamics based on the Beddoes-Leishman model. A two state (per calculation point) model of dynamic inflow will be included.



Nonlinear kinematic formulation

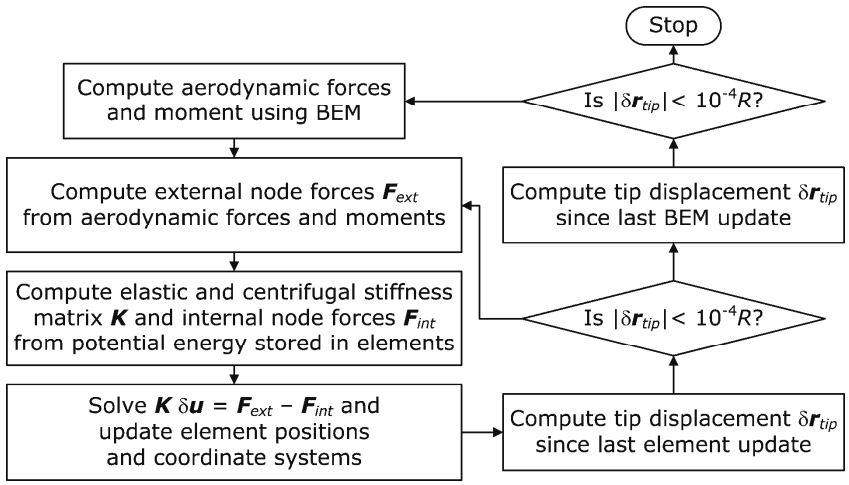


Element positions and orientations

$$\mathbf{r}_{e,k} = \sum_{n=1}^{k-1} \mathbf{T}_{e,n} \left(\delta \mathbf{u}_n + (l_n + \Delta l_n) \mathbf{e}_3 \right)$$

$$\mathbf{T}_{e,k} = \left(\prod_{n=1}^{k-1} \mathbf{E}_n \mathbf{T} \left[\delta \boldsymbol{\theta}_n \right] \right) \mathbf{E}_k$$

Nonlinear steady state



Open-loop aeroelastic equations

Structural mass
Structural damping
Gyroscopic forces
Aerodynamic damping
Elastic and centrifugal stiffness
Geometric stiffness of steady state force
Aerodynamic stiffness
Coupling to aerodynamic states

$$\mathbf{M}\ddot{\mathbf{x}}_s + (\mathbf{C} + \mathbf{G} + \mathbf{C}_a)\,\dot{\mathbf{x}}_s + (\mathbf{K} + \mathbf{K}_{sf} + \mathbf{K}_a)\,\mathbf{x}_s + \mathbf{A}_f\mathbf{x}_a = \mathbf{F}_s$$

$$\dot{\mathbf{x}}_a + \mathbf{A}_d\mathbf{x}_a + \mathbf{C}_{sa}\dot{\mathbf{x}}_s + \mathbf{K}_{sa}\mathbf{x}_s = \mathbf{F}_a$$

Time lags

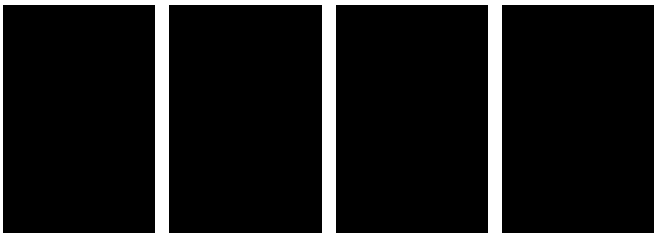
Coupling to structural states

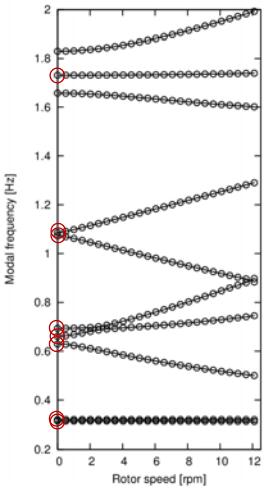
$\mathbf{x}_s$ = elastic and bearing degrees of freedom
$\mathbf{x}_a$ = aerodynamic state variables
$\mathbf{F}_s, \mathbf{F}_a$ = forces due to actuators and wind disturbance

Standstill modes of the 5MW NREL turbines

Mode 1 | Mode 2 | Mode 3 | Mode 4

Mode 5 | Mode 6 | Mode 7 | Mode 9





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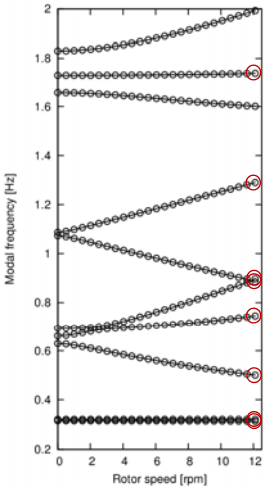
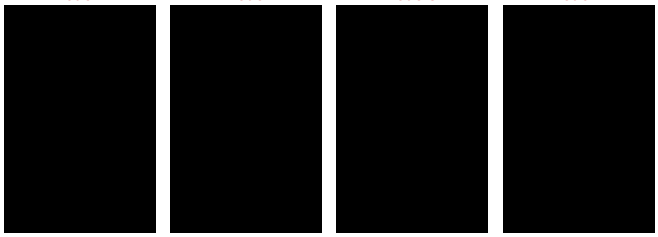
Structural modes during operation



Mode 1 | Mode 2 | Mode 3 | Mode 4



Mode 5 | Mode 6 | Mode 7 | Mode 9

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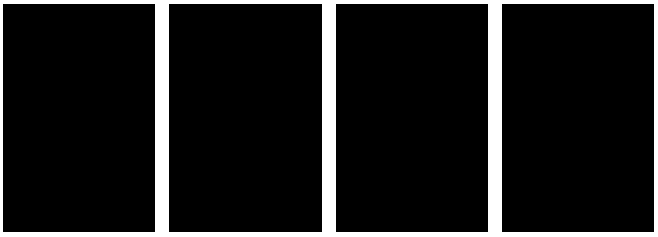
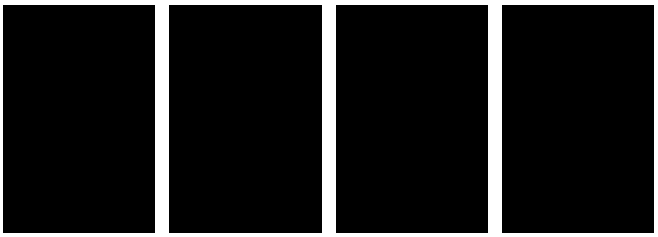
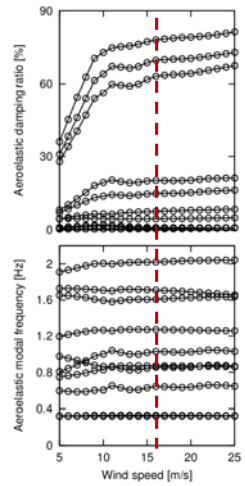
Open-loop aeroelastic modes

Mode 1 | Mode 2 | Mode 3 | Mode 4



Mode 5 | Mode 6 | Mode 7 | Mode 9





Closed-loop aero-servo-elastic equations

Open-loop aeroelastic system in first order form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_{\mathrm{v}}\mathbf{v}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{D}_{\mathrm{v}}\mathbf{v}$$

Controller equations

$$\dot{\mathbf{x}}_c = \mathbf{A}_c\mathbf{x}_c + \mathbf{B}_c\mathbf{y}$$
$$\mathbf{u} = \mathbf{C}_c\mathbf{x}_c + \mathbf{D}_c\mathbf{y}$$

Closed-loop equations

$$\dot{\mathbf{x}}_{\mathrm{ase}} = \mathbf{A}_{\mathrm{ase}}\mathbf{x}_{\mathrm{ase}} + \mathbf{B}_{\mathrm{ase}}\mathbf{u}_{\mathrm{perb}} + \mathbf{B}_{v,\mathrm{ase}}\mathbf{v}$$
$$\mathbf{y}_{\mathrm{ase}} = \mathbf{C}_{\mathrm{ase}}\mathbf{x}_{\mathrm{ase}} + \mathbf{D}_{\mathrm{ase}}\mathbf{u}_{\mathrm{perb}} + \mathbf{D}_{v,\mathrm{ase}}\mathbf{v}$$

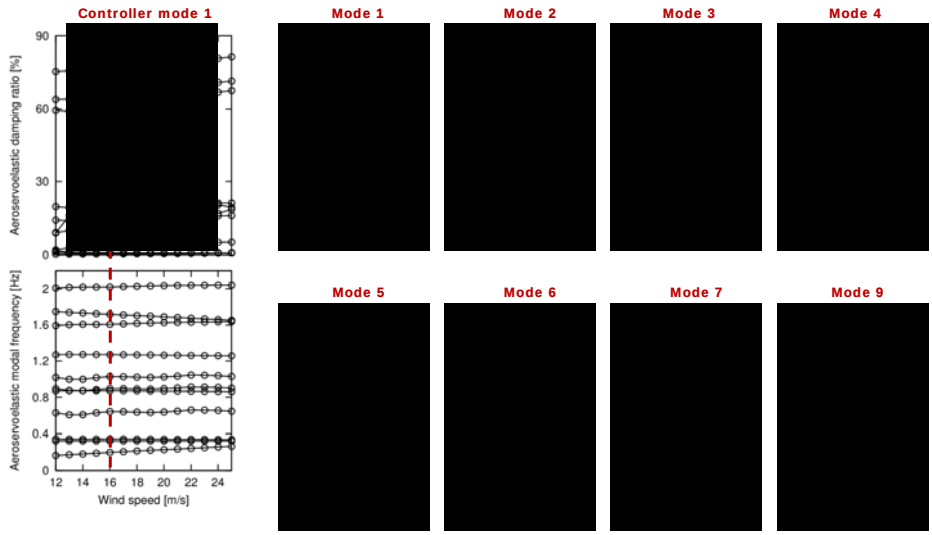
Wind disturbances as linear shears

$$\mathbf{v} = \begin{Bmatrix} v_{\mathrm{col}} \\ v_{\mathrm{ver}} \\ v_{\mathrm{hor}} \end{Bmatrix}$$

or as local axial flow velocities

$$\mathbf{v} = \begin{Bmatrix} \mathbf{v}_{\mathrm{col}} \\ \mathbf{v}_{\mathrm{cos}} \\ \mathbf{v}_{\mathrm{sin}} \end{Bmatrix}$$

Closed-loop aero-servo-elastic modes



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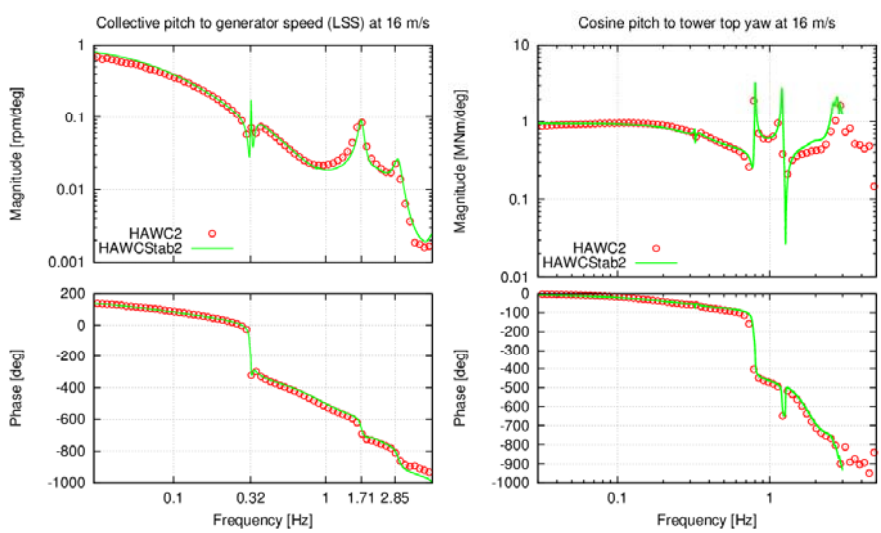
HAWCStab2 validation with HAWC2: Linear vs. nonlinear frequency responses



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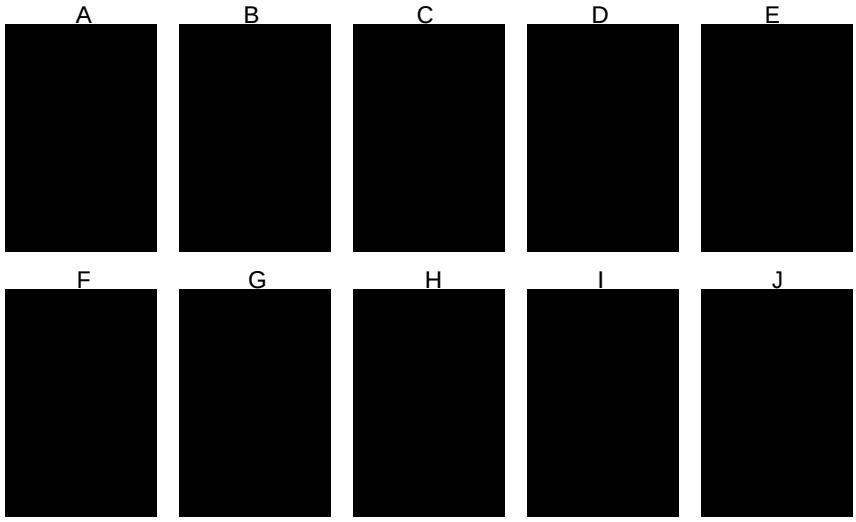
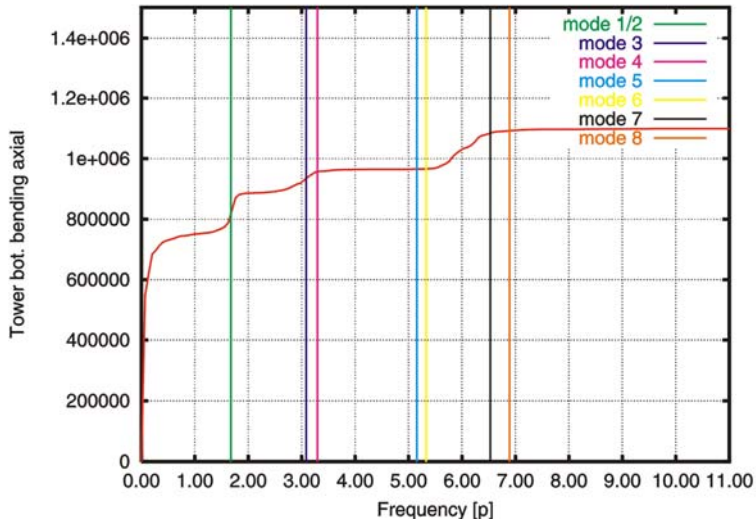
MODAL ANALYSIS

Group work: Name these modes and order them after frequencies



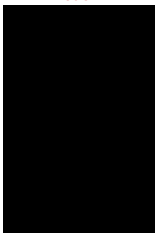
Why consider modal dynamics of wind turbines?

Accumulated load spectrum for 1.4 MW turbine

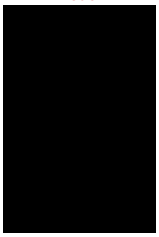


Standstill modes of the 5MW NREL turbines

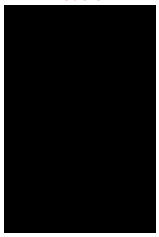
Mode 1



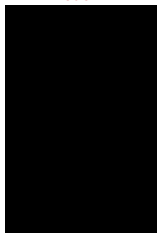
Mode 2



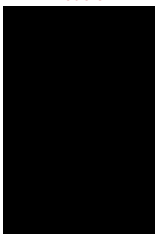
Mode 3



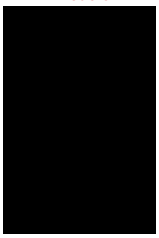
Mode 4



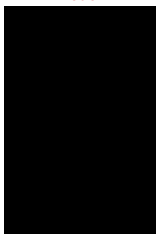
Mode 5



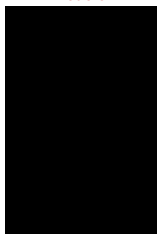
Mode 6

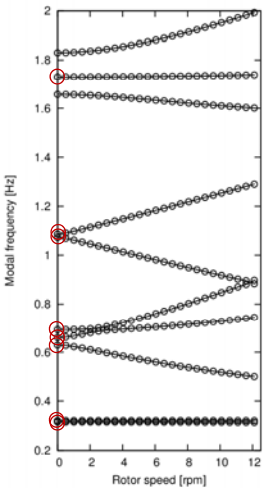


Mode 7



Mode 9





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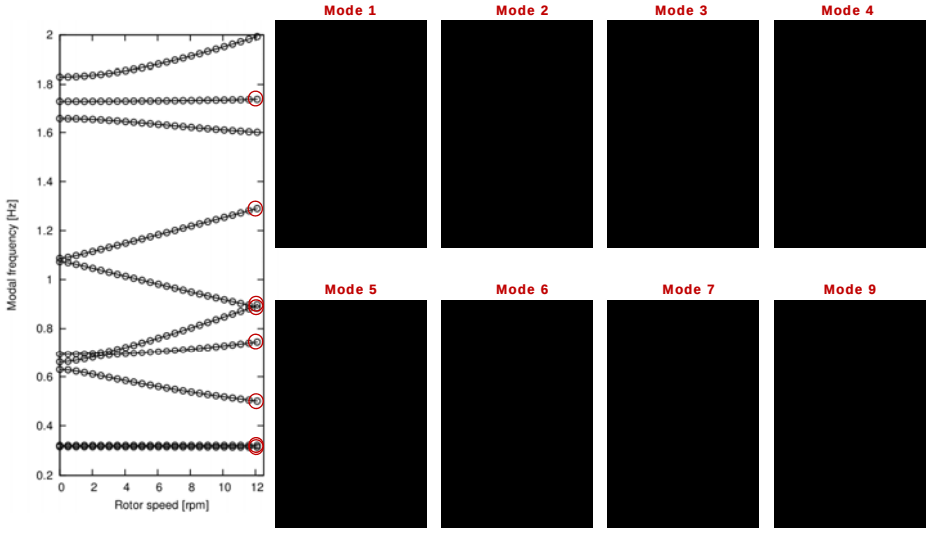
What happens when the rotor is rotating?

- Frequencies and mode shapes depend on the frame of reference (ground fixed or rotor fixed) of the observer
  - Backward and forward whirling rotor modes
  - Concept of period mode shapes
  - Multiple resonance frequencies of a single mode

- Vibration analysis are complicated by time-varying system matrices
  - Direct eigenvalue analysis is nonsense!
  - Floquet theory for stability analysis
  - Lyapunov-Floquet (L-F) transformation for eigenvalue analysis of time-invariant system
  - **Coleman transformation** as a special case of L-F transformation for three and more bladed rotors
  - Implicit Floquet theory, Hill's method and perturbation methods

Structural modes during operation

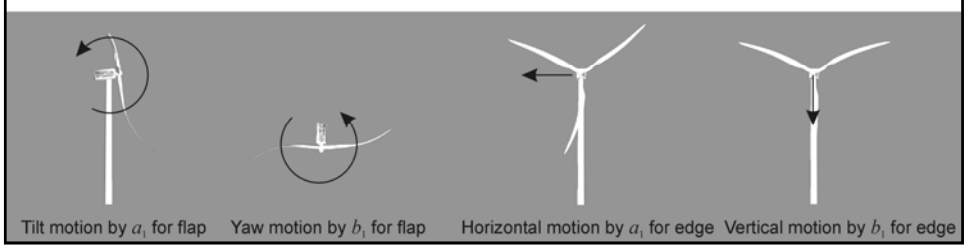


Coleman transformation into multi-blade coordinates

Physical blade coordinates described by multi-blade coordinates

$$
\begin{aligned}
q_k(t) &= a_0(t) + a_1(t)\cos\psi_k + b_1(t)\sin\psi_k \\
&= a_0(t) \\
&+ a_1(t)\cos\left(\Omega t + \frac{2\pi}{3}(k-1)\right) \\
&+ b_1(t)\sin\left(\Omega t + \frac{2\pi}{3}(k-1)\right)
\end{aligned}
$$

All blade coordinates transformed to the ground fixed frame



Modal response measured on blade

The eigenvalue solution $\mathbf{v}e^{\lambda t} = \mathbf{v}e^{(\alpha + i\omega)t}$ for the multi-blade coordinates

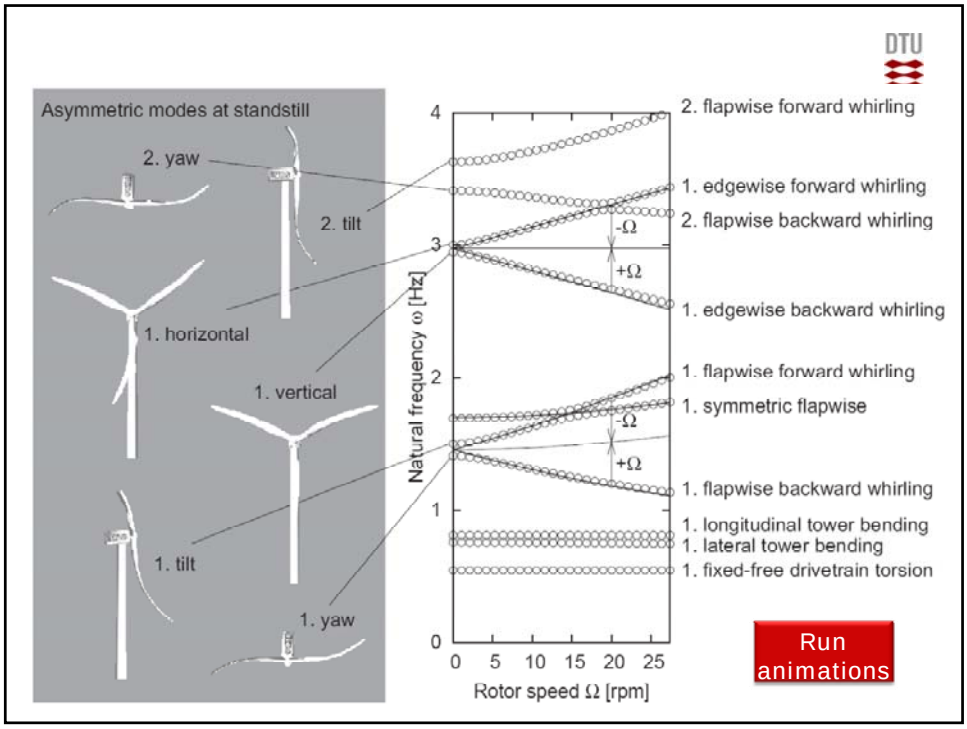
$$
\begin{aligned}
a_0(t) &= e^{\alpha t}\left(\mathrm{Re}\{v_{a_0}\}\cos(\omega t) - \mathrm{Im}\{v_{a_0}\}\sin(\omega t)\right) \\
a_1(t) &= e^{\alpha t}\left(\mathrm{Re}\{v_{a_1}\}\cos(\omega t) - \mathrm{Im}\{v_{a_1}\}\sin(\omega t)\right) \\
b_1(t) &= e^{\alpha t}\left(\mathrm{Re}\{v_{b_1}\}\cos(\omega t) - \mathrm{Im}\{v_{b_1}\}\sin(\omega t)\right)
\end{aligned}
$$

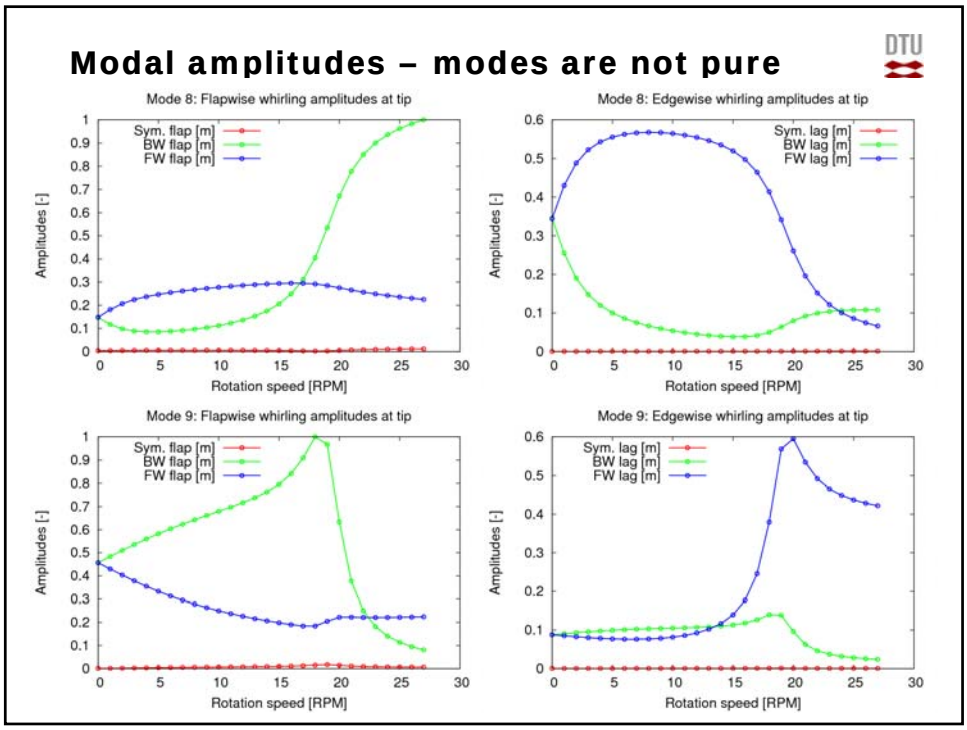
Insertion into Coleman transformation yields response in the rotor frame:

$$
\begin{aligned}
q_k &= e^{\alpha t}\left(A_0\cos(\omega t + \phi_0)\right. \\
&+ \tfrac{1}{2}A_{\mathrm{BW}}\cos\left((\omega+\Omega)t + \frac{2\pi}{3}(k-1) + \phi_{\mathrm{BW}}\right) \\
&+ \tfrac{1}{2}A_{\mathrm{FW}}\cos\left(\left.(\omega-\Omega)t - \frac{2\pi}{3}(k-1) + \phi_{\mathrm{FW}}\right)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
A_0 &= \tfrac{1}{2}\sqrt{\mathrm{Re}\{v_{a_0}\}^2 + \mathrm{Im}\{v_{a_0}\}^2} \\
A_{\mathrm{BW}} &= \tfrac{1}{2}\sqrt{(\mathrm{Re}\{v_{b_1}\} - \mathrm{Im}\{v_{a_1}\})^2 + (\mathrm{Im}\{v_{b_1}\} + \mathrm{Re}\{v_{a_1}\})^2} \\
A_{\mathrm{FW}} &= \tfrac{1}{2}\sqrt{(\mathrm{Re}\{v_{b_1}\} + \mathrm{Im}\{v_{a_1}\})^2 + (\mathrm{Re}\{v_{a_1}\} - \mathrm{Im}\{v_{b_1}\})^2}
\end{aligned}
$$

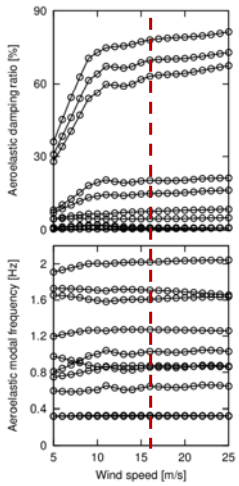

DTU
Asymmetric modes at standstill
2. yaw
2. tilt
1. horizontal
1. vertical
1. tilt
1. yaw
Natural frequency ω [Hz]
Rotor speed Ω [rpm]
2. flapwise forward whirling
1. edgewise forward whirling
2. flapwise backward whirling
1. edgewise backward whirling
1. flapwise forward whirling
1. symmetric flapwise
1. flapwise backward whirling
1. longitudinal tower bending
1. lateral tower bending
1. fixed-free drivetrain torsion
-Ω
+Ω
Run animations


Modal amplitudes – modes are not pure
DTU
Mode 8: Flapwise whirling amplitudes at tip
Mode 8: Edgewise whirling amplitudes at tip
Mode 9: Flapwise whirling amplitudes at tip
Mode 9: Edgewise whirling amplitudes at tip
Sym. flap [m]
BW flap [m]
FW flap [m]
Sym. lag [m]
BW lag [m]
FW lag [m]
Amplitudes [-]
Rotation speed [RPM]

Open-loop aeroelastic modes

Mode 1 | Mode 2 | Mode 3 | Mode 4

Mode 5 | Mode 6 | Mode 7 | Mode 9



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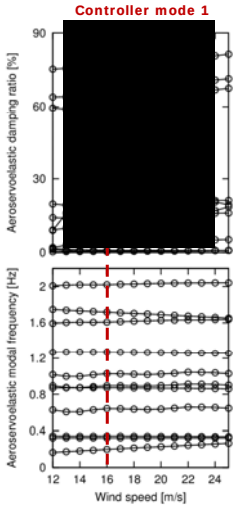
Closed-loop aero-servo-elastic modes

Controller mode 1



Mode 1 | Mode 2 | Mode 3 | Mode 4

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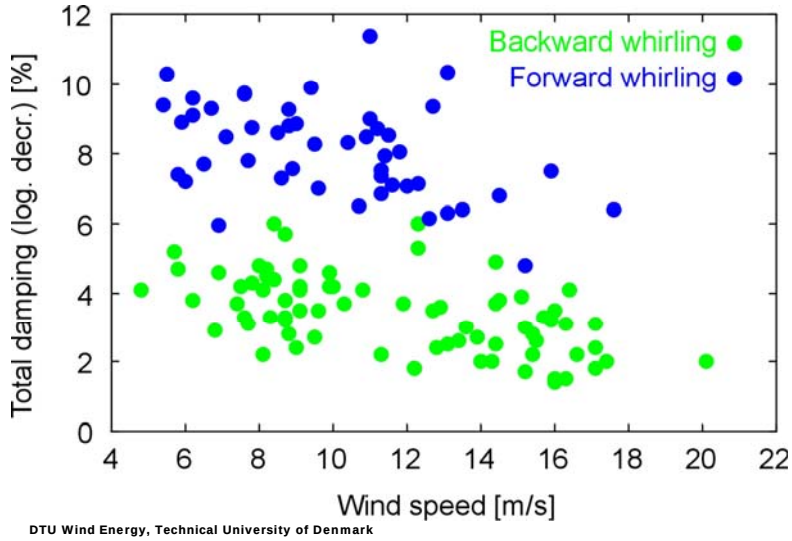
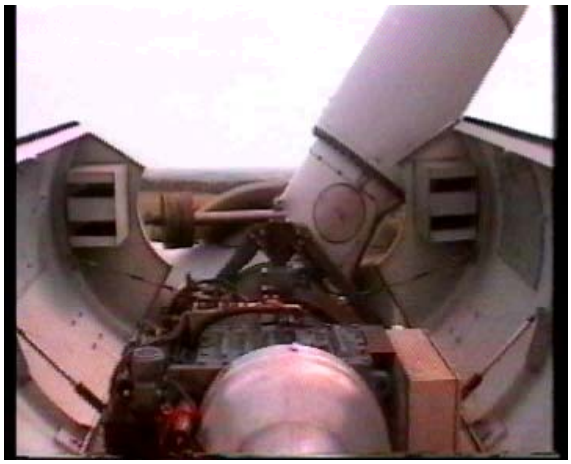
Validation of HAWCStab(1) against meaurements

STALL-REGULATED TURBINE

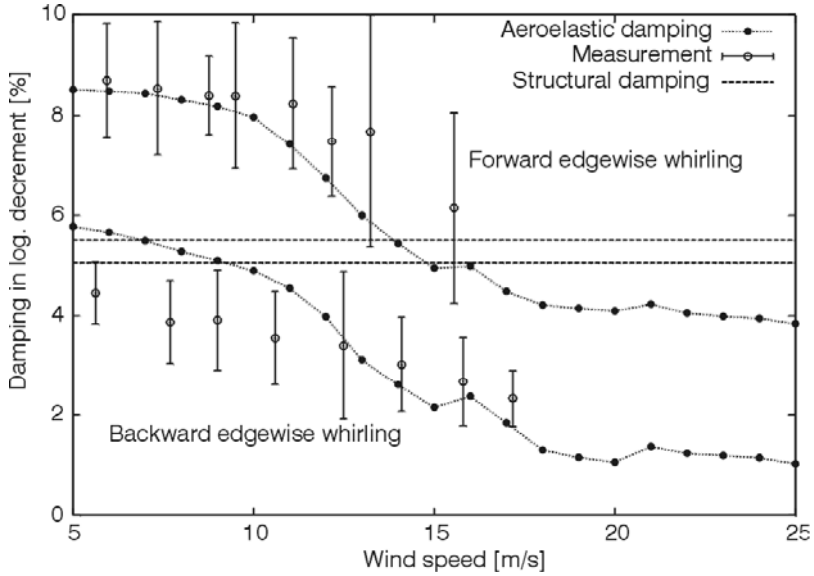
Damping of edgewise whirling modes from experiments



Experimental setup



Predictions versus measurements

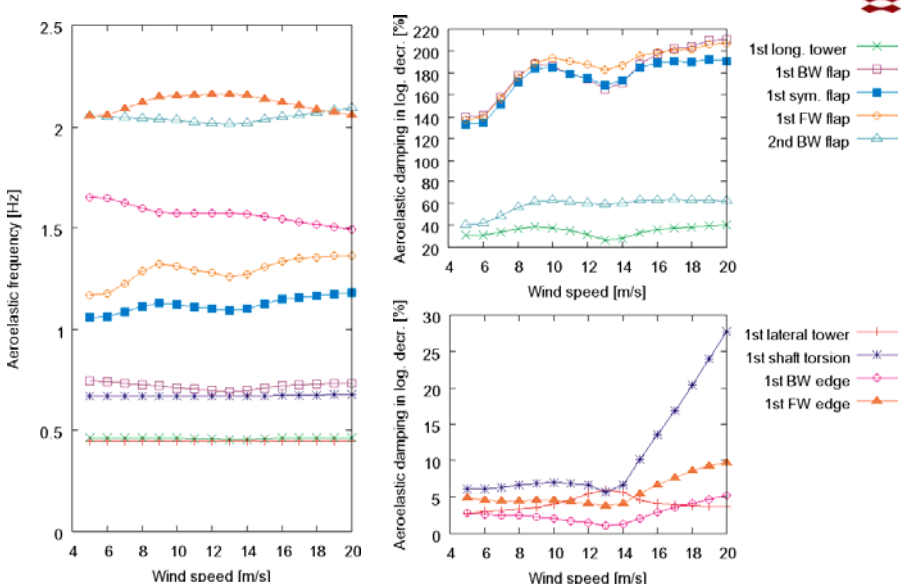


NM80 prototype in Tjæreborg

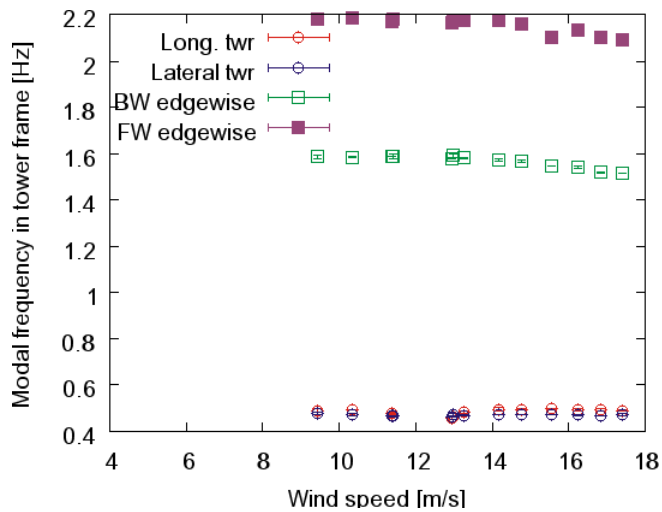
- Prototype only
- PRVS turbine
- 2750 kW rated power
- Hub height 57 m
- Rotor diameter 80 m
- Electric pitch actuators

Too large for mechanical exciter, e.g. tower excitation needs 1 ton on 1 m arm

HAWCStab: Aeroelastic modes



Modal frequencies from system identification



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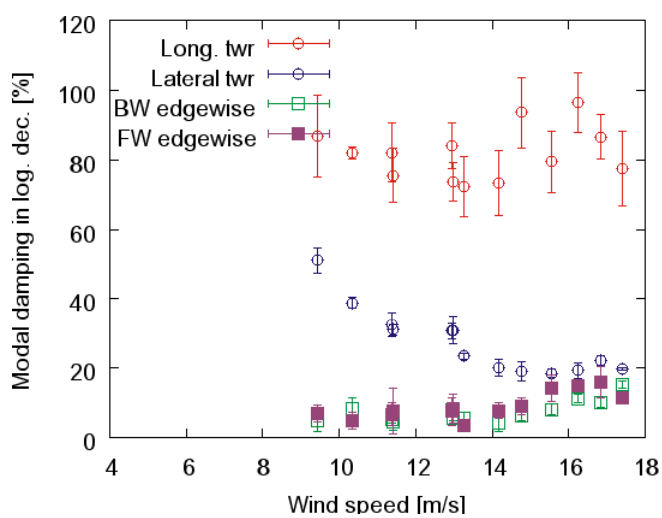
Modal damping from system identification



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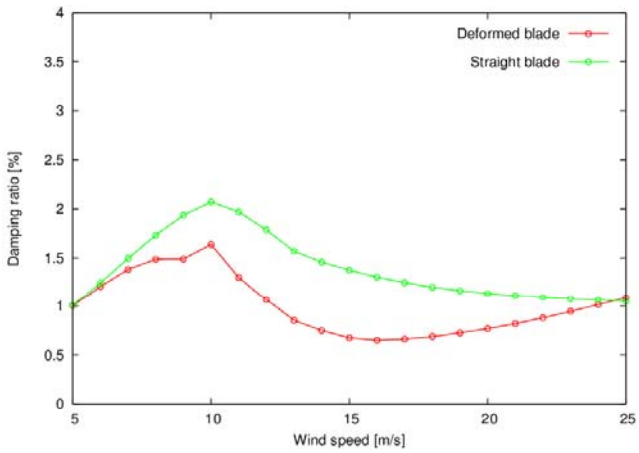
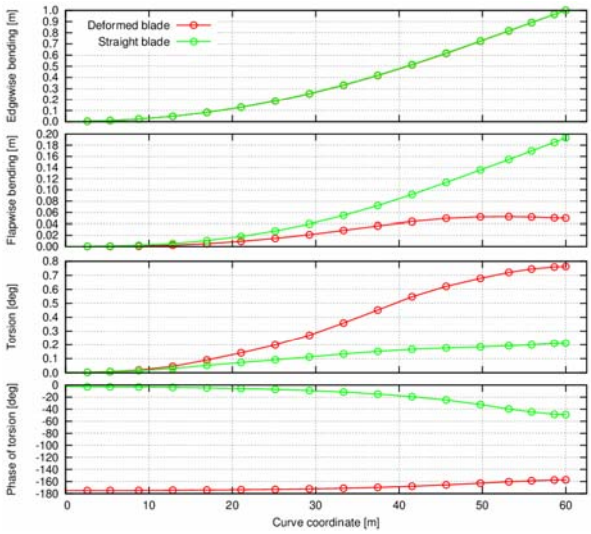
Blades with large flapwise deflections

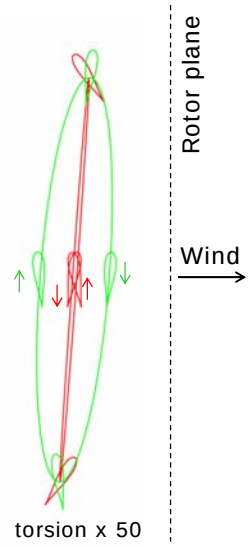
EFFECT OF TORSION ON EDGEWISE DAMPING

Aeroelastic edgewise damping of NREL blade



Torsional content in edgewise mode due to flapwise bending at 15 m/s

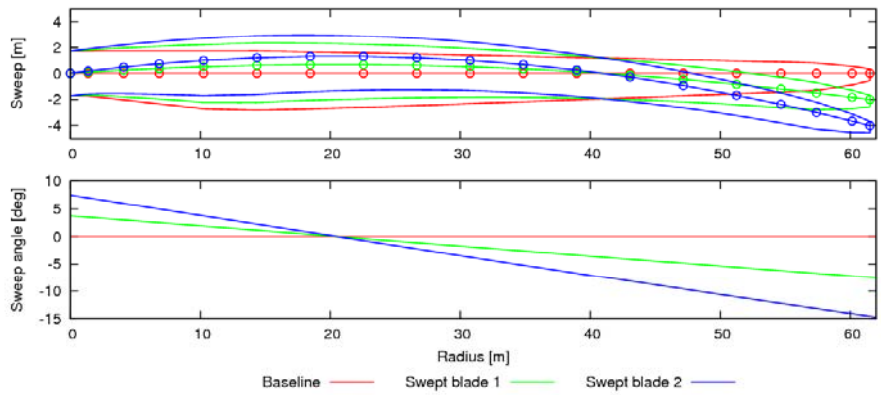




Another example of bend-twist couplings

BACKWARD SWEPT BLADES

Backward swept blades
Baseline – NREL 61.5m with CG at EA



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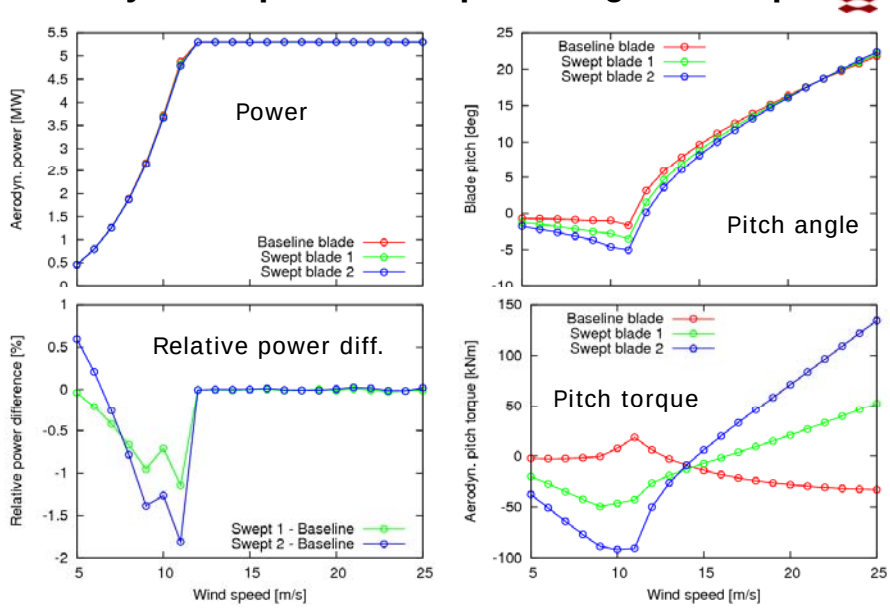
Steady state power and pitch angle & torque



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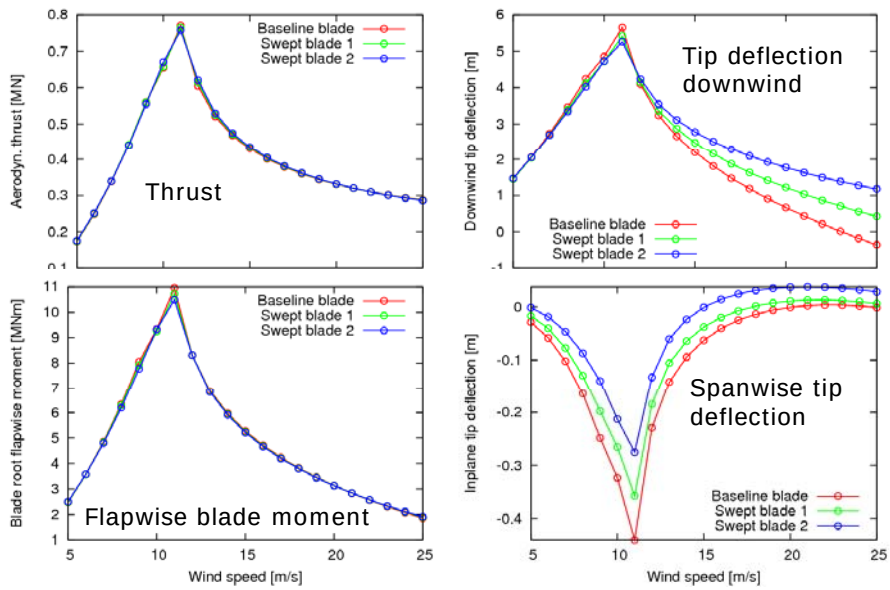
Steady state thrust and tip deflection



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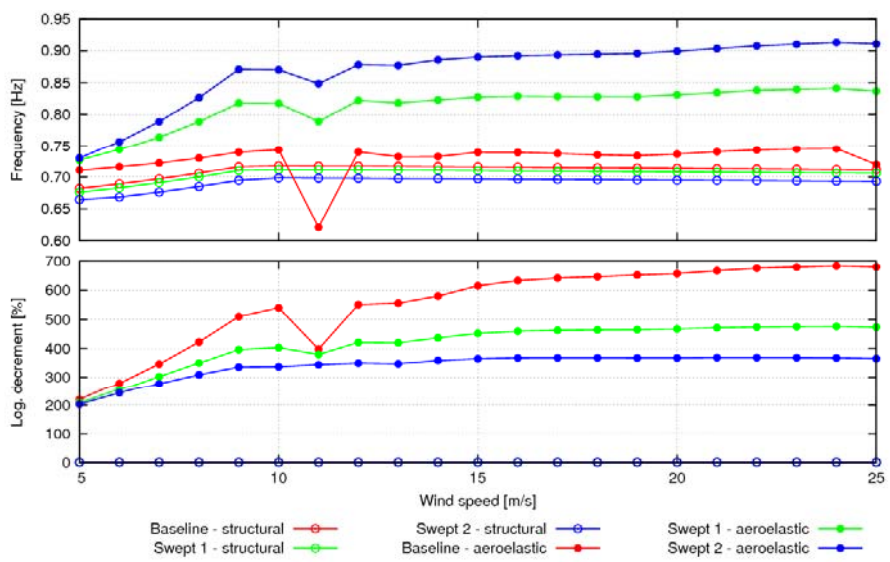
Modal frequencies and damping – 1st flap



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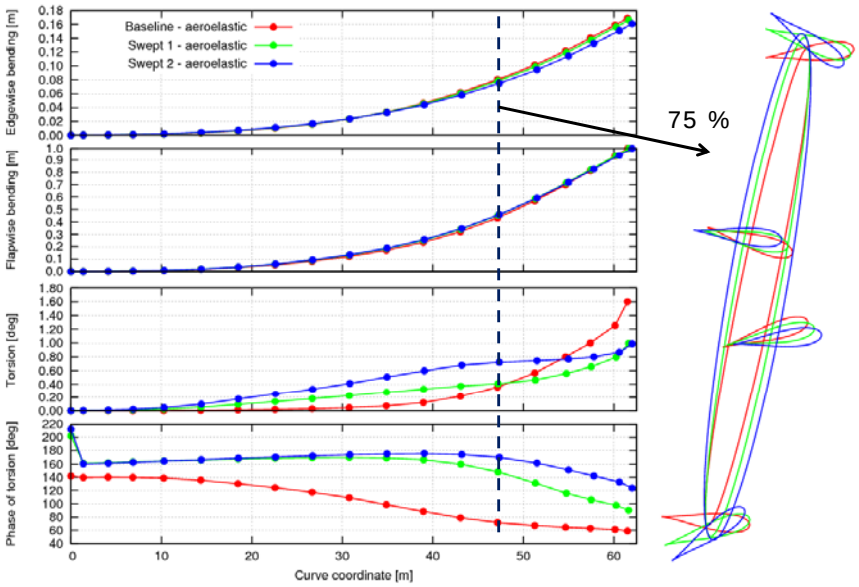
Aeroelatic flapwise mode shape at 10 m/s



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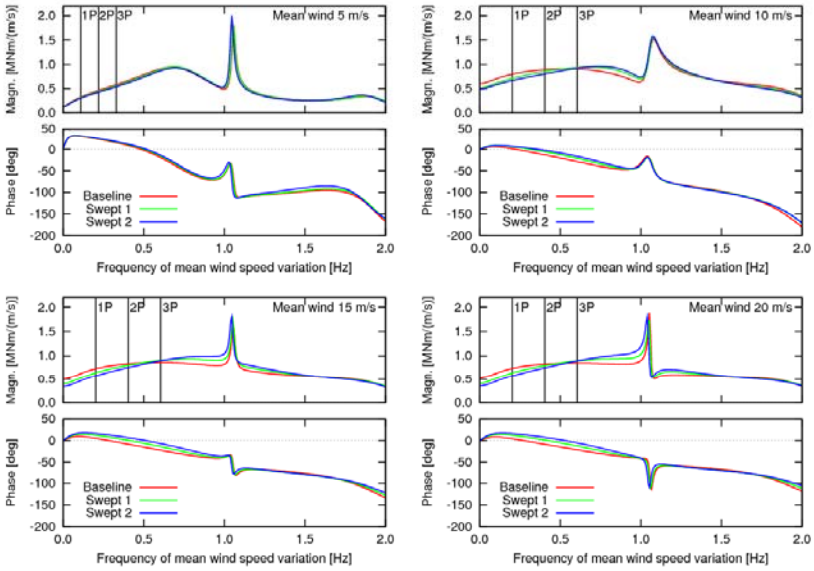
Flapwise blade root moment



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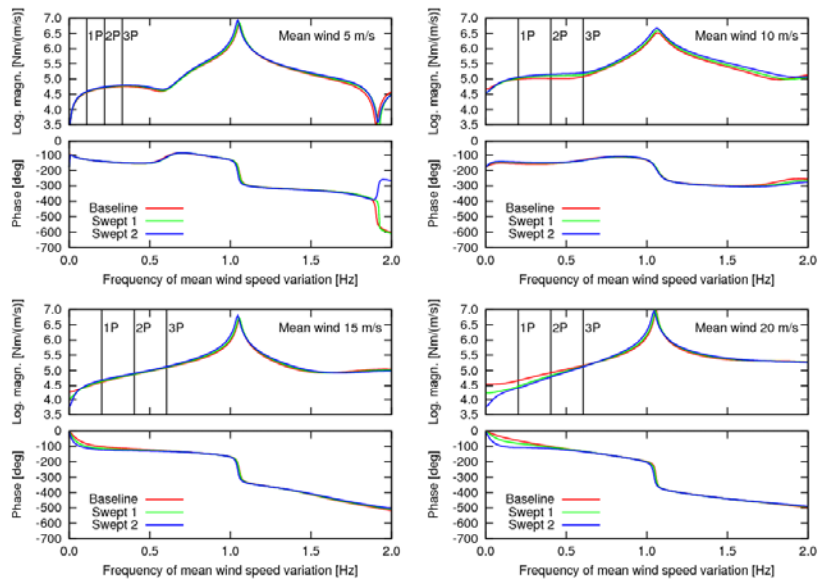
Edgewise blade root moment



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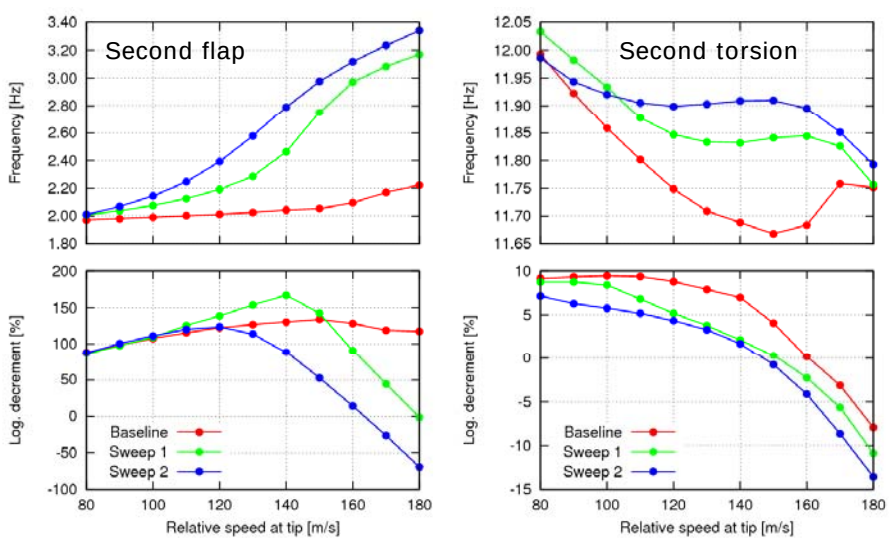
Lowest damped modes for 0 deg pitch and increasing relative speed ($\lambda = \sqrt{99}$)



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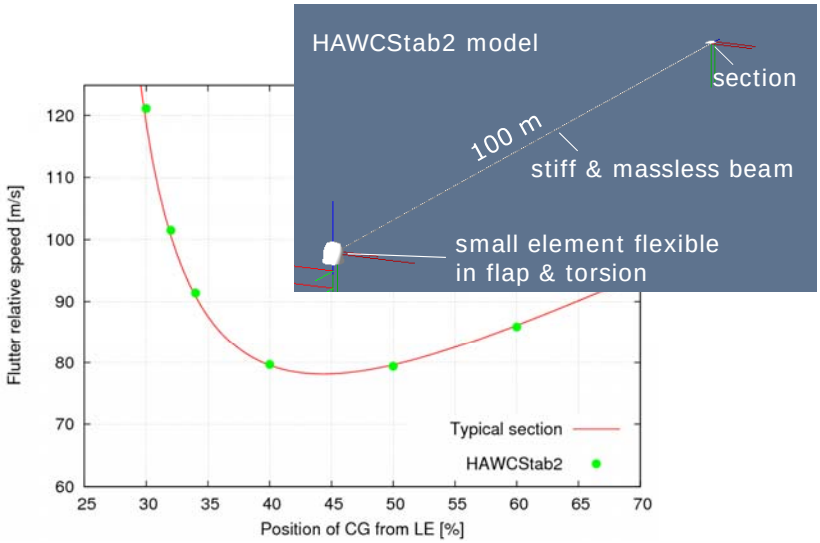
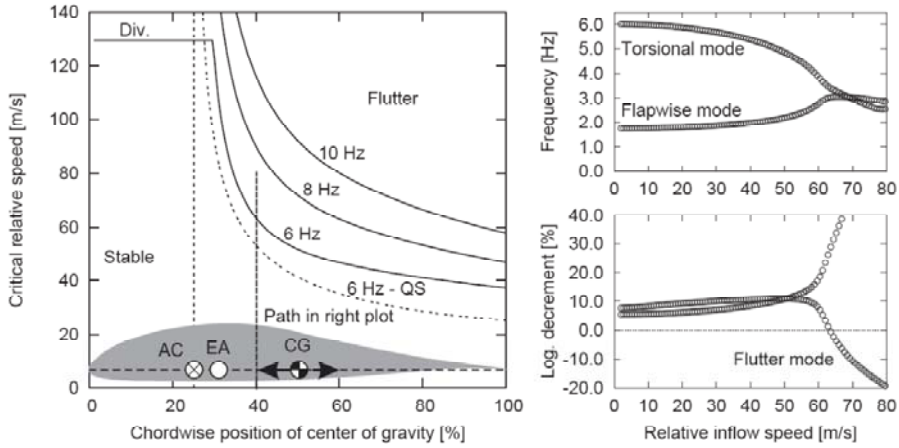
FLUTTER ANALYSIS

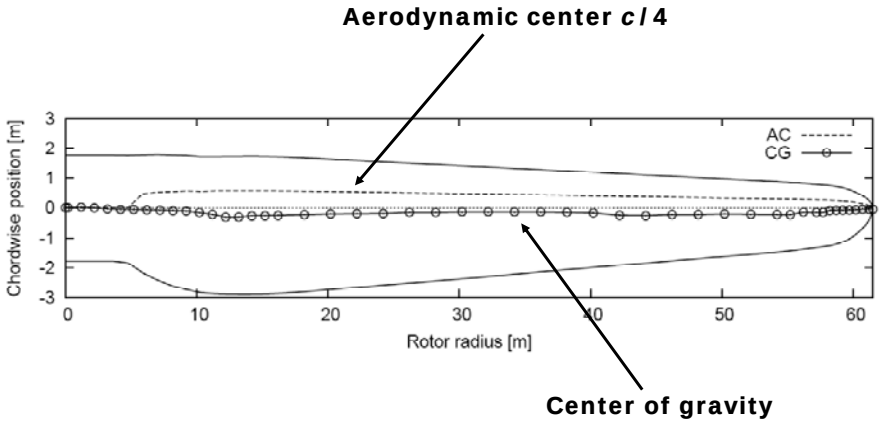
Validation against Theodorsen results



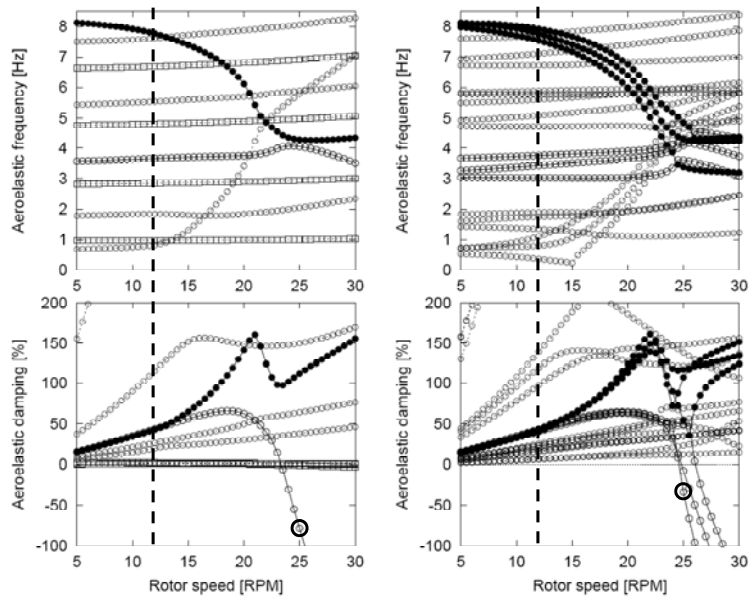
Flutter limit on relative speed



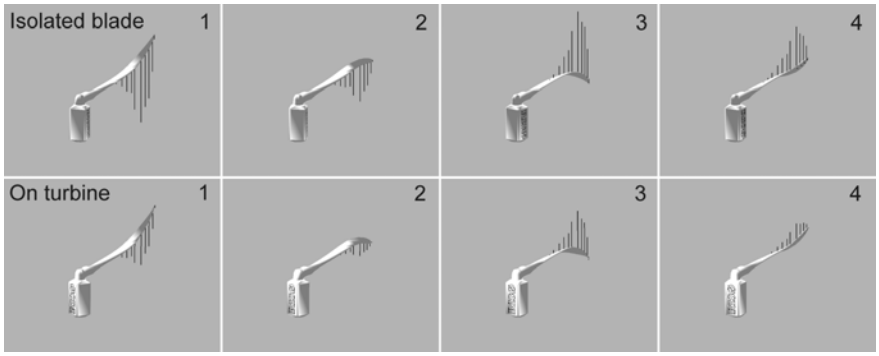
Blade planform of NREL 5 MW



Flutter limit on rotor speed

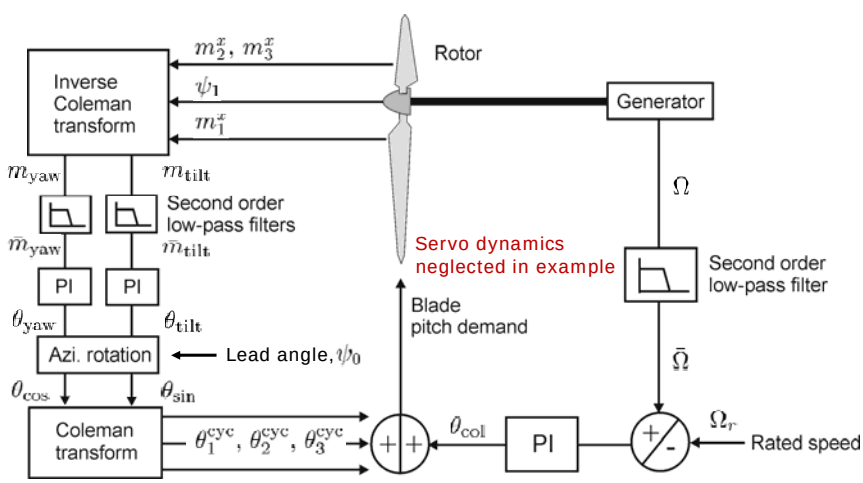


Flutter mode shape



TUNING OF PITCH CONTROLLER

Collective and cyclic pitch controllers



Tuning of collective pitch controller

Linear closed-loop equation for small drivetrain motion $\Omega = \Omega_r + \dot{\phi}$

$$\underbrace{\left(I_r + n_g^2 I_g\right)}_{\text{mass}} \ddot{\phi} + \underbrace{\left(\frac{1}{\eta}\frac{\partial Q_g}{\partial \Omega}\bigg|_0 - \cancelto{0}{\frac{\partial Q}{\partial \Omega}\bigg|_0} - \frac{\partial Q}{\partial \theta}\bigg|_0 k_P\right)}_{\text{damping}} \dot{\phi} - \underbrace{\frac{\partial Q}{\partial \theta}\bigg|_0 k_I}_{\text{stiffness}} \phi = 0$$

Tuning by placement of closed-loop drivetrain $\lambda = -\zeta_\Omega \omega_\Omega \pm i\omega_\Omega$

$$k_P = \frac{2\,\zeta_\Omega \omega_\Omega \left(I_r + {n_g}^2 I_g\right) - \frac{1}{\eta}\frac{\partial Q_g}{\partial \Omega}\Big|_0}{-\frac{\partial Q}{\partial \theta}\Big|_0} \quad \text{where } \frac{\partial Q_g}{\partial \Omega}\bigg|_0 = \begin{cases} 0 & \text{const. torque} \\ -\frac{P_r}{\Omega_r^2} & \text{const. power} \end{cases}$$

$$k_I = \frac{\omega_\Omega^2 (1 + \zeta_\Omega^2)\left(I_r + n_g^2 I_g\right)}{-\frac{\partial Q}{\partial \theta}\Big|_0}$$

Pole choice for NREL 5MW RWT:
$\omega_\Omega = 0.1$ Hz and $\zeta_\Omega \in [0.6 : 0.7]$

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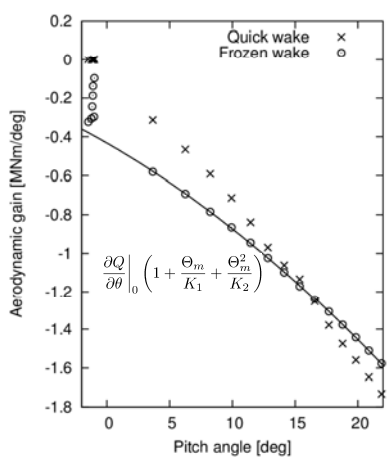
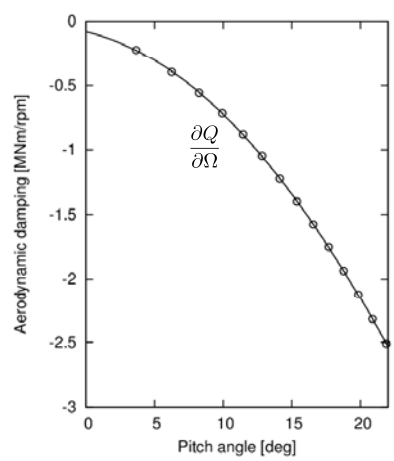
Aerodynamic gain and damping (frozen wake)





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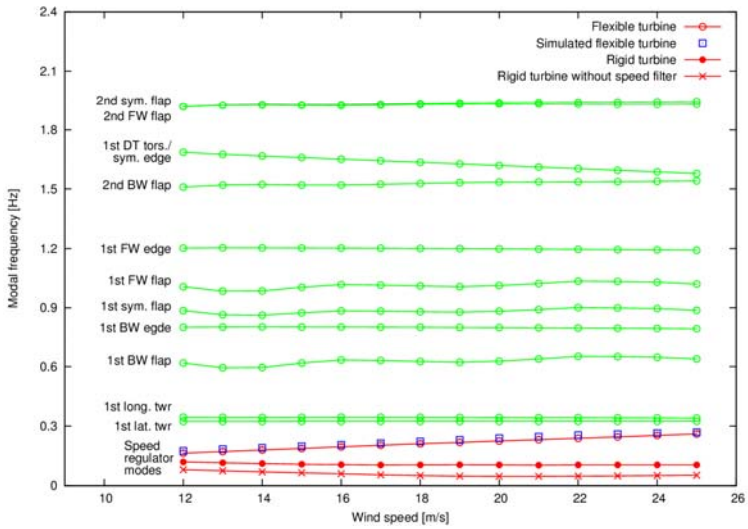
Closed-collective-pitch-loop modal frequencies



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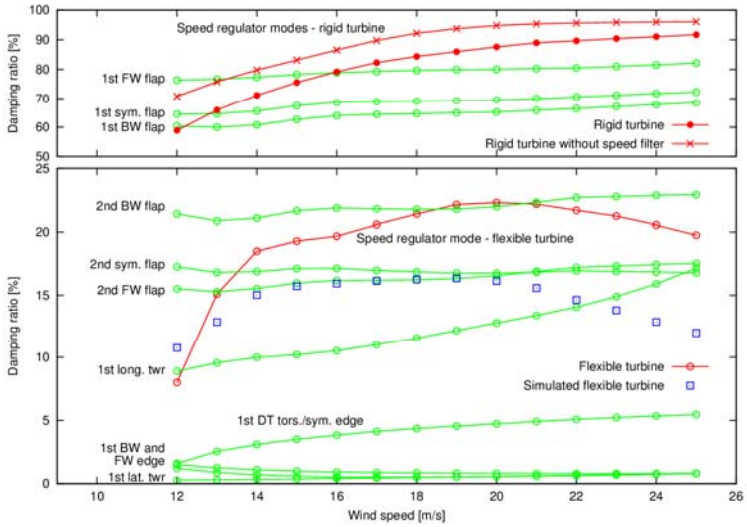
Closed-collective-pitch-loop modal damping



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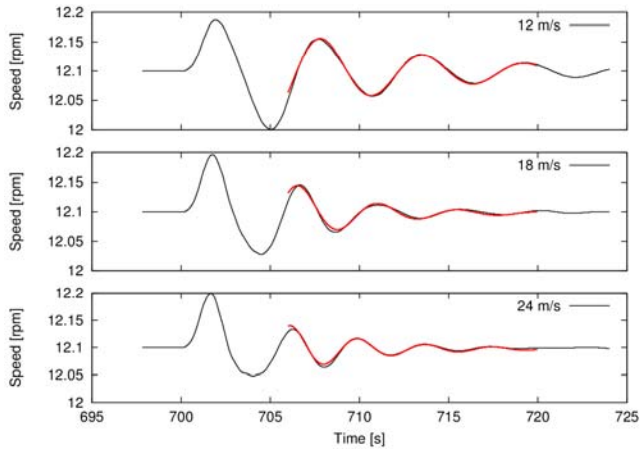
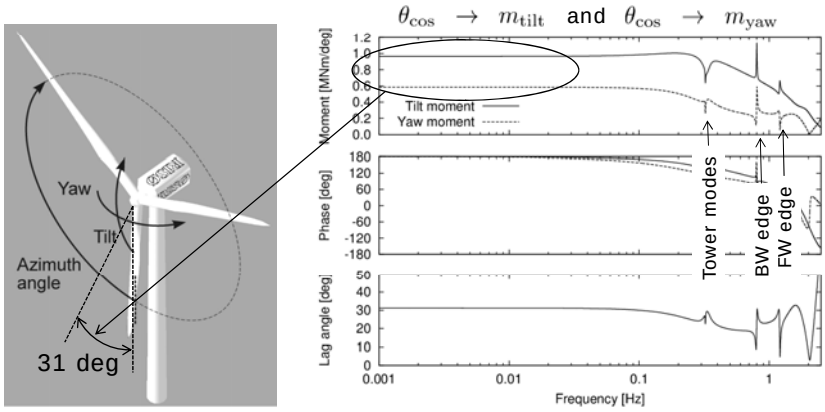
Pole estimations using HAWC2 simulations

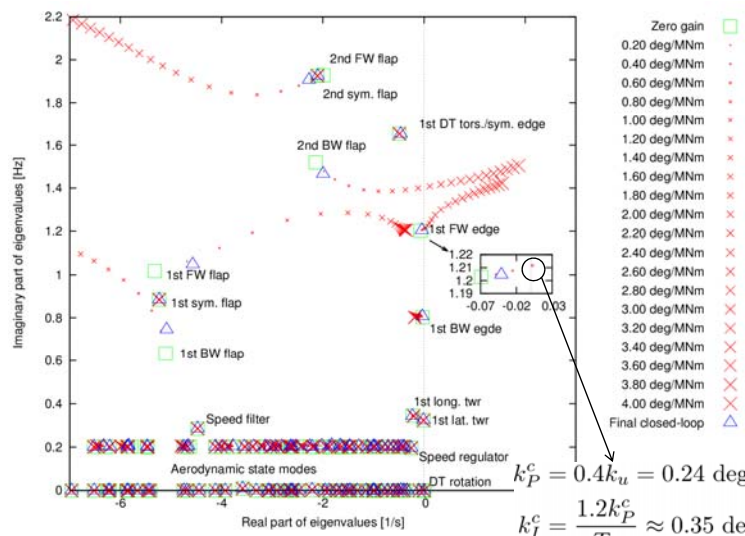


Tuning of cyclic pitch controller at 16 m/s - Lead angle from open-loop analysis



+ the phase of the low-pass filters at 1P

Cyclic gains by Ziegler-Nichols method - pole plot in case of 25P filter and 16 m/s



$$k_P^c = 0.4k_u = 0.24 \text{ deg/MNm}$$

$$k_I^c = \frac{1.2k_P^c}{T_u} \approx 0.35 \text{ deg/s/MNm}$$

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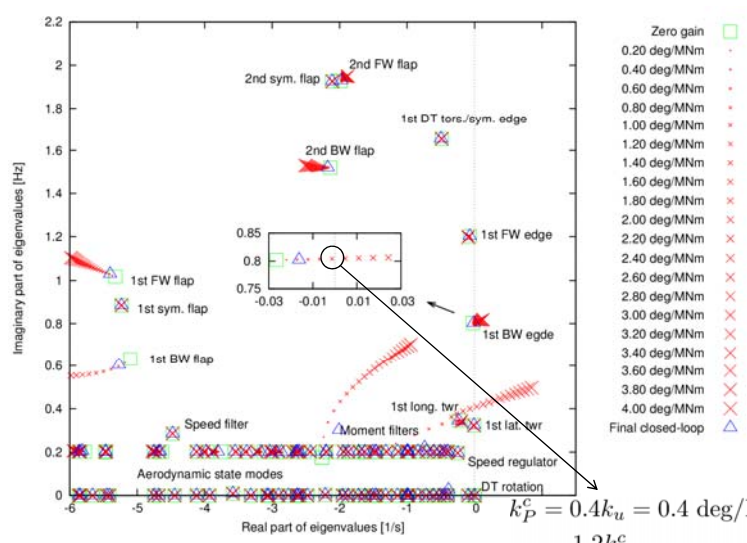
Cyclic gains by Ziegler-Nichols method - pole plot in case of 2P filter and 16 m/s



$$k_P^c = 0.4k_u = 0.4 \text{ deg/MNm}$$

$$k_I^c = \frac{1.2k_P^c}{T_u} \approx 0.39 \text{ deg/s/MNm}$$

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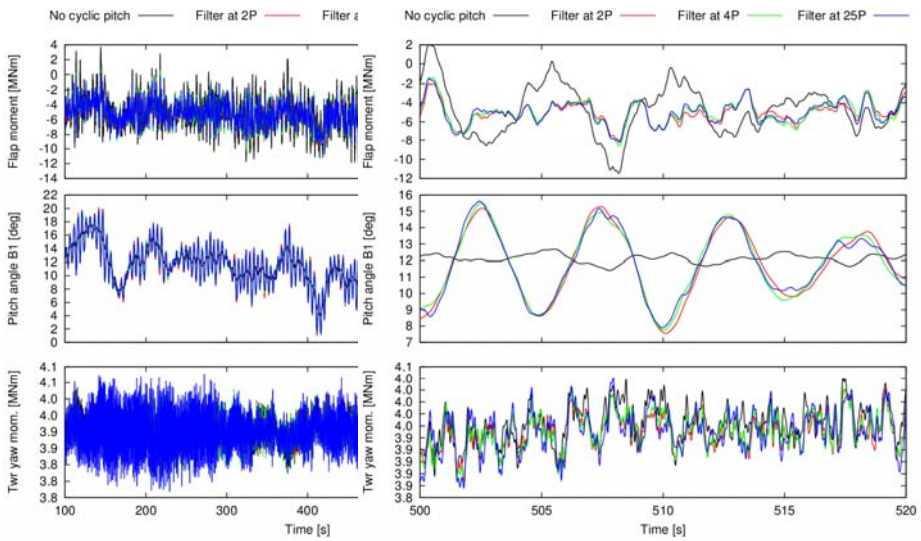
HAWC2 simulations at 16 m/s with NTM



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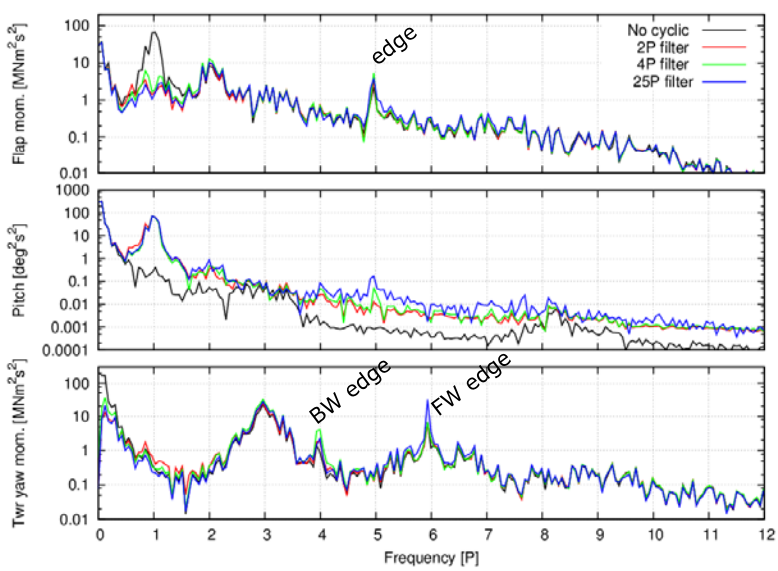
Response power spectra from simulations



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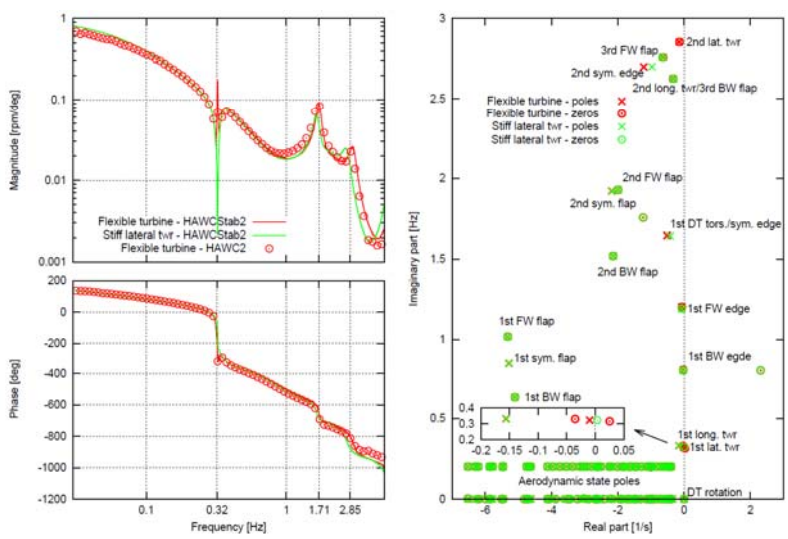
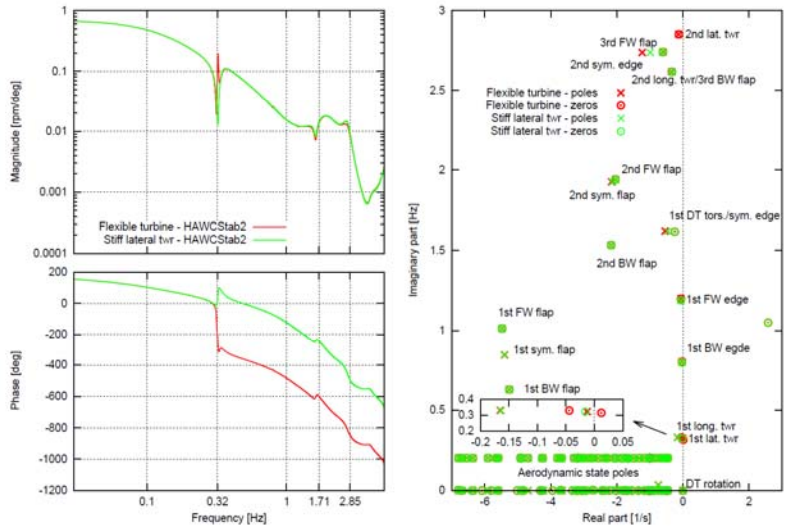
POLES AND ZEROS IN T.F. FROM PITCH TO SPEED

Operation of NREL 5MW RWT at 16 m/s



Operation of NREL 5MW RWT at 20 m/s



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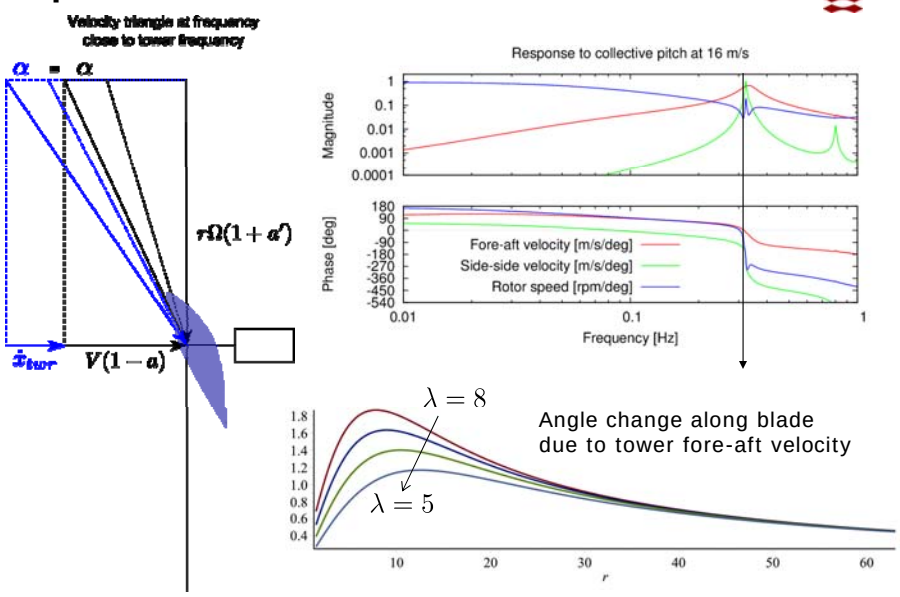
Explanation of this zero



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Summary

- HAWCStab2 can be used for performing open-loop and closed-loop eigenvalue and frequency-domain analysis of three-bladed turbines:
  - Controller equations can be user-defined.
  - Full order analyses can be performed both inside or outside HAWCStab2 by writing out system matrices for each operation point.
  - Reduced order modelling capabilities are currently performed outside HAWCStab2. Automated procedures for obtaining models with desired details will be implemented in HAWCStab2, or in Matlab scripts.
- HAWCStab2 is a common tool for both control engineers and mechanical engineers:
  - It can provide first-principle models for model-based controllers.
  - It can explain phenomena observed in load simulations.