HAWC2 course

Modeling of Vertical Axis Wind Turbines (VAWT's) in HAWC2

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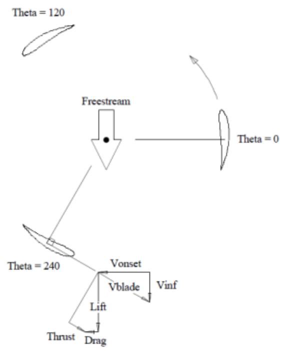
The VAWT principle



J. Vassberg, A.K. Gopinath, A. Jameson . Revisiting the Vertical-Axis Wind-Turbine Design using Advanced Computational Fluid Dynamics. AIAA Paper 2005-0047, 43rd AIAA ASM, Reno

Renewed interest in Vertical Axis Wind Turbines

Designs of the 1980´s

Recent designs

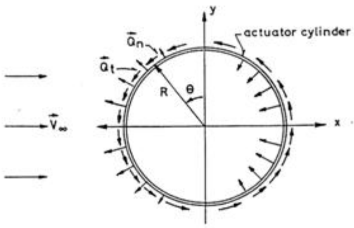
Credits Image by Grimshaw & Wind Power Ltd

Photo: Grimshaw Architects.

2D solution of the flowfield

- The actuator cylinder model

The AC model was developed from 1979 to 1982 in a PhD study by Madsen. The basic idea behind the model is to extend the well-known AD concept for HAWT´s to a general approach of an actuator surface coinciding with the swept area of the actual turbine. For a straight bladed VAWT the swept surface is cylindrical and this is the surface geometry the model has been developed for in its present form. Further, the model is in a 2D version in order to limit the complexity of the model and thus make it suitable for implementation in an aeroelastic model but the general model is described in a 3D version.



The basic equations

Now the Euler Equations take the form:

$$\frac{\partial w_x}{\partial x} + w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_x}{\partial y} = -\frac{\partial p}{\partial x} + f_x$$

$$\frac{\partial w_y}{\partial x} + w_x \frac{\partial w_y}{\partial x} + w_y \frac{\partial w_y}{\partial y} = -\frac{\partial p}{\partial y} + f_y$$

and the equation of continuity

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0$$

A linear solution

$$w_x = -\frac{1}{2\pi}\int_0^{2\pi} Q_r(\varphi)\frac{-(x+\sin\varphi)\sin\varphi+(y-\cos\varphi)\cos\varphi}{(x+\sin\varphi)^2+(y-\cos\varphi)^2}d\varphi$$
$$-Q_r(\arccos(y))^* + Q_r(-\arccos(y))^{**}$$

$$w_y = -\frac{1}{2\pi}\int_0^{2\pi} Q_r(\theta)\frac{-(x+\sin\varphi)\cos\varphi-(y-\cos\varphi)\sin\varphi}{(x+\sin\varphi)^2+(y-\cos\varphi)^2}d\varphi$$

Assuming that the loading is piece wise linear, the equations can be rewritten in discrete form

$$w_x = \sum_{i=1}^{i=N} Q_{r,i}\int_{\varphi_i-\frac{1}{2}\Delta\varphi}^{\varphi_i+\frac{1}{2}\Delta\varphi} \frac{-(x+\sin\varphi)\cos\varphi-(y-\cos\varphi)\sin\varphi}{(x+\sin\varphi)^2+(y-\cos\varphi)^2}d\varphi$$

$$w_y = \sum_{i=1}^{i=N} Q_{r,i}\int_{\varphi_i-\frac{1}{2}\Delta\varphi}^{\varphi_i+\frac{1}{2}\Delta\varphi} \frac{-(x+\sin\varphi)\cos\varphi-(y-\cos\varphi)\sin\varphi}{(x+\sin\varphi)^2+(y-\cos\varphi)^2}d\varphi$$

Which can be reformulated

$$w_{x,j} = \sum_{i=1}^{i=N} Q_{r,i} R_{wx,i,j} - Q^*_{r,j} + Q^*_{r,(N-j)}$$

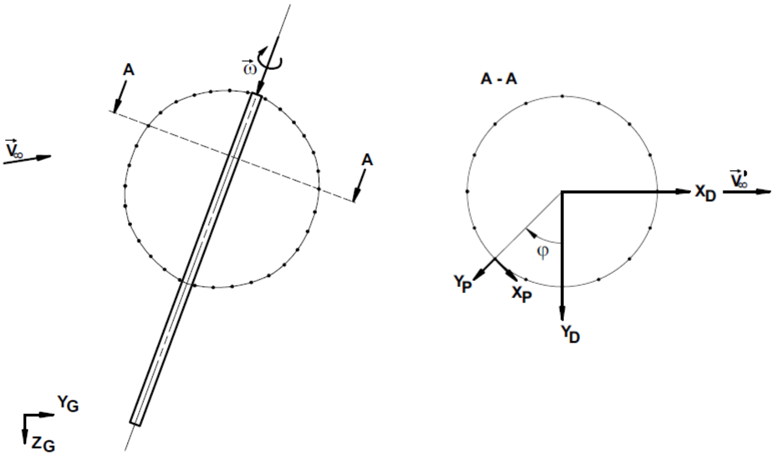
$$w_{y,j} = \sum_{i=1}^{i=N} Q_{r,i} R_{wy,i,j}$$

Which the influence coefficients R can be calculated initially once and for all

$$R_{wx,j,i} = \int_{\varphi_i - \frac{1}{2}\Delta\varphi}^{\varphi_i + \frac{1}{2}\Delta\varphi} \frac{-(x+\sin\varphi)\cos\varphi - (y-\cos\varphi)\sin\varphi}{(x+\sin\varphi)^2 + (y-\cos\varphi)^2} d\varphi$$

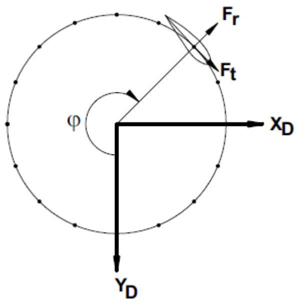
$$R_{wy,j,i} = \int_{\varphi_i - \frac{1}{2}\Delta\varphi}^{\varphi_i + \frac{1}{2}\Delta\varphi} \frac{-(x+\sin\varphi)\cos\varphi - (y-\cos\varphi)\sin\varphi}{(x+\sin\varphi)^2 + (y-\cos\varphi)^2} d\varphi$$

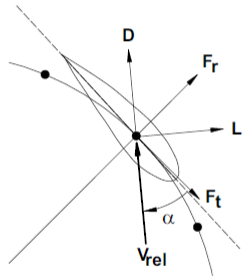
Implementation in HAWC2



Wind is projected to a 2D coordinate system aligned perpendicular to the shaft and with local x along the average wind direction.

Local forces in the 2D system



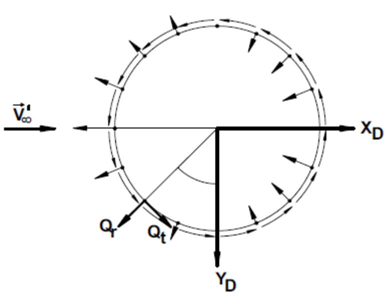


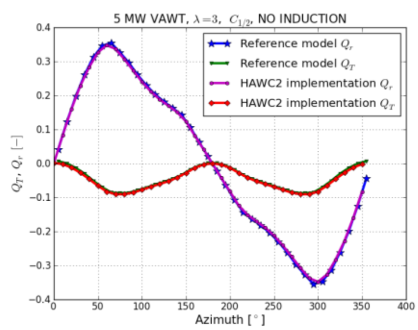
$$\mathbf{C}_P = \mathbf{T}_{GP}\mathbf{T}_{LG}\begin{bmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} C_d \\ C_l \\ 0 \end{Bmatrix}$$

$$F_r = \frac{1}{2}\rho c V_r^2 \mathbf{C}_P(2) \qquad F_t = \frac{1}{2}\rho c V_r^2 \mathbf{C}_P(1)$$

$$C_T = \frac{T}{\frac{1}{2}\rho A V_\infty^2}$$

$$C_T = \frac{\frac{1}{2\pi R}\int_0^{2\pi}(F_n(\varphi)\sin\varphi - F_t(\varphi)\cos\varphi)NRd\varphi}{\frac{1}{2}\rho A V_\infty^2}$$





$$Q_r = \frac{NF_r}{2\pi R\rho V_\infty^2}$$

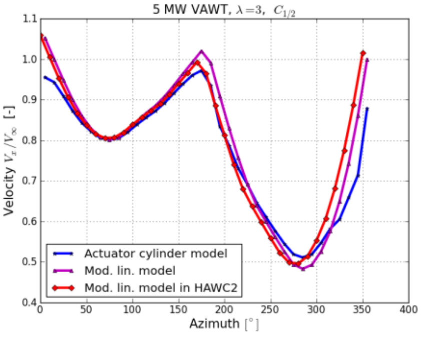
$$Q_t = \frac{NF_t}{2\pi R\rho V_\infty^2}$$

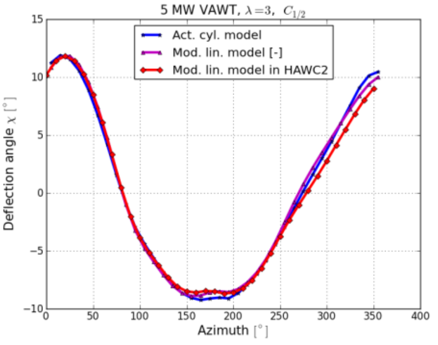
which leads to

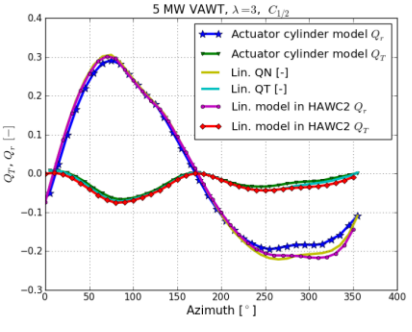
$$Q_r = \frac{-V_r^2\mathbf{C}_P(2)cN}{4\pi R V_\infty^2}$$

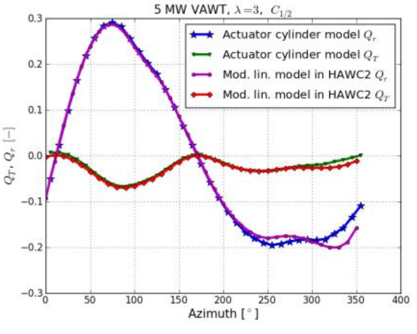
$$Q_t = \frac{V_r^2\mathbf{C}_P(1)cN}{4\pi R V_\infty^2}$$

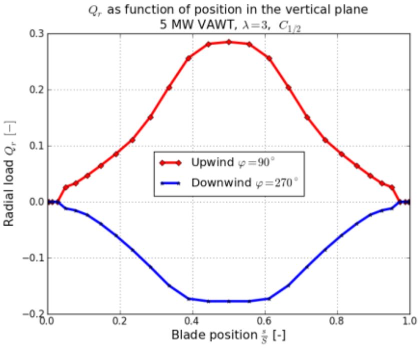
Comparison with induction

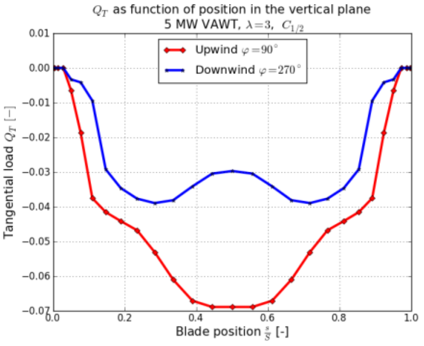






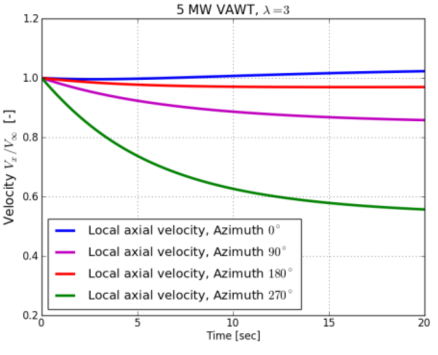




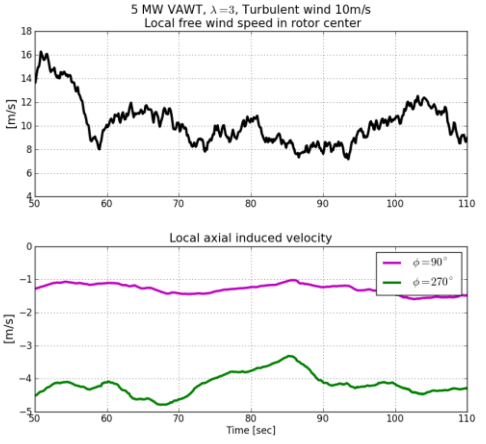


Dynamic inflow effects

- A similar approach as for a HAWT has been used to account for dynamic lag of induced velocities



Turbulence



HTC input

```
begin main_body;
   name        blade1 ;
   type        timoschenko ;
   nbodies      1;
   node_distribution     c2_def ;
   damping   0.84 0.826 0.28 0.0023 0.002 0.002 ;
   begin timoschenko_input;
     filename ./data/hawc2_st-5MW-VAWT.003 ;
     set 1 6 ;              set subset   stiff
   end timoschenko_input;
     begin          c2_def          ;
                    nsec            27              ;
                    sec             1               0.0000          0.0005          0.0000          0.0000          ;
                    sec             2               0.0000          13.2999         5.0000          0.0000          ;
                    sec             3               0.0000          24.6518         10.0000         0.0000          ;
                    sec             4               0.0000          34.2099         15.0000         0.0000          ;
                    sec             5               0.0000          42.1280         20.0000         0.0000          ;
                    sec             6               0.0000          48.5599         25.0000         0.0000          ;
                    sec             7               0.0000          53.6593         30.0000         0.0000          ;
                    .
                    .
                    .
                    sec             22              0.0000          48.5599         105.0000        0.0000          ;
                    sec             23              0.0000          42.1280         110.0000        0.0000          ;
                    sec             24              0.0000          34.2099         115.0000        0.0000          ;
                    sec             25              0.0000          24.6518         120.0000        0.0000          ;
                    sec             26              0.0000          13.2999         125.0000        0.0000          ;
                    sec             27              0.0000          0.0005          130.0000        0.0000          ;
     end            c2_def          ;
 end main_body;
```

HTC input

```
Begin aero;
    nblades  2;
    hub_vec floater 3 2 ; rotor vector through  the VAWT rotor
    Link 1 mbdy_c2_def blade1;
    link 2 mbdy_c2_def blade2;
    ae_filename        ./data/hawc_ae_5MW_VAWT.002 ;
    pc_filename        ./data/hawc_pc-5MW_VAWT.002 ;
    induction_method   3 ;    0=none, 1=normal,2=near wake, 3=vawt
    aerocalc_method    1 ;    0=ingen aerodynamic, 1=med aerodynamic
    aerosections       29 ;
    ae_sets          3 3 ;
    tiploss_method     01;    0=none, 1=normal
    dynstall_method    02;    0=none, 1=stig øye method,2=mhh method
    output_profile_coef_filename x1 ;
end aero ;
```