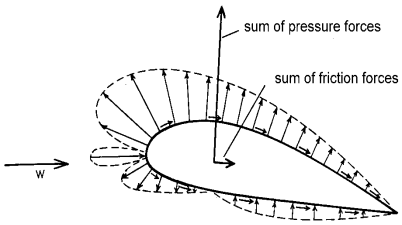
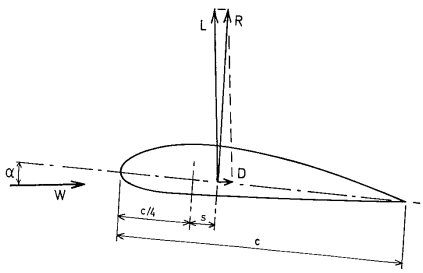
HAWC2 course

Lesson 2b: Aerodynamics

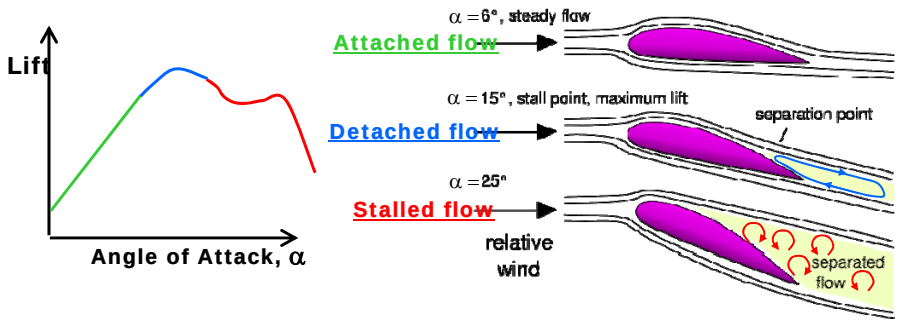
Risø DTU
National Laboratory for Sustainable Energy

Lift and drag





Stall



Stall types

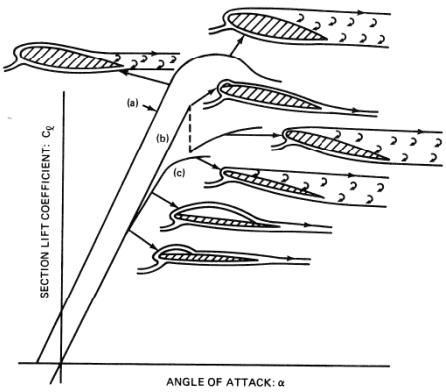
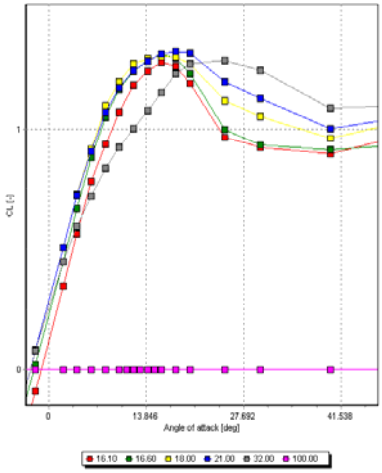
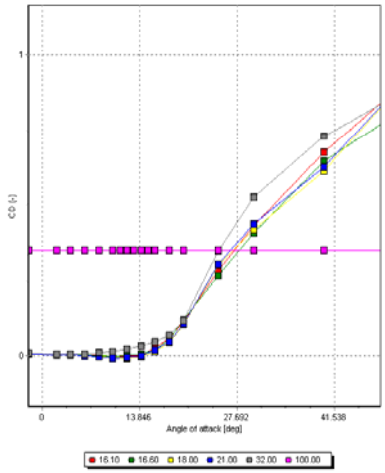


Figure 6.18 Three types of stall: (a) trailing-edge, (b) leading-edge, and (c) thin-airfoil

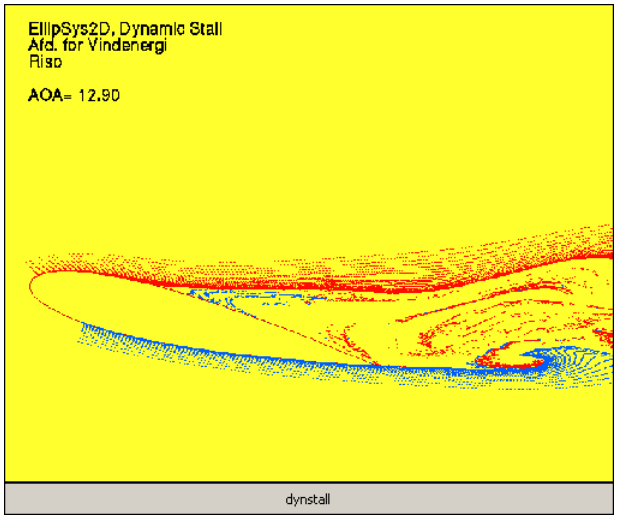
Lift and drag – example



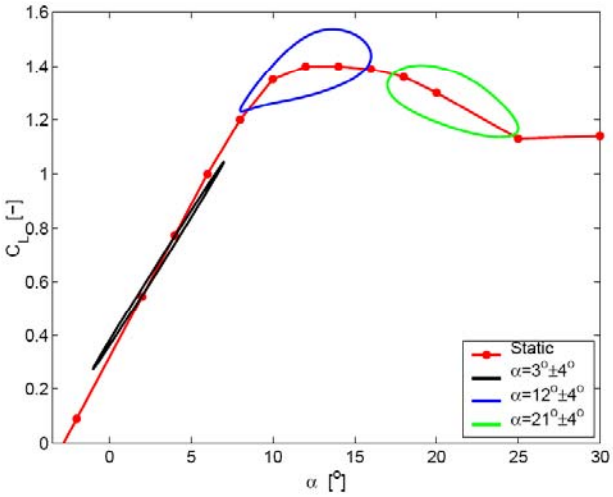


$$L = \tfrac{1}{2}\rho W^2 c C_L(\alpha) \qquad D = \tfrac{1}{2}\rho W^2 c C_D(\alpha)$$

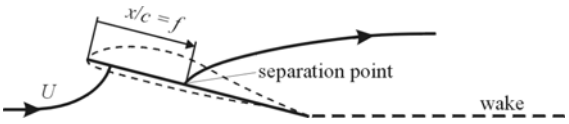
Results from CFD: Dynamic stall

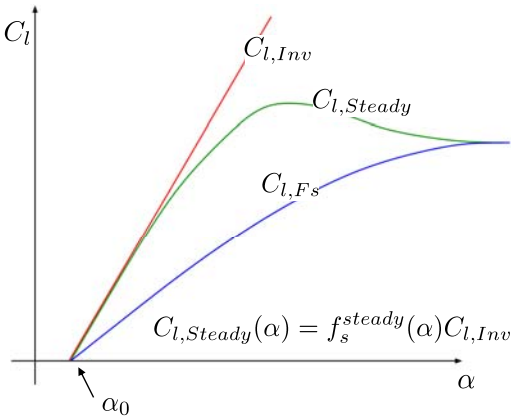


Dynamic stall effects



The Øye model





$$C_{l,Steady}(\alpha) = f_s^{steady}(\alpha) C_{l,Inv} + [1 - f_s^{steady}(\alpha)] C_{l,Fs}(\alpha)$$

Determining the separation function

- The separation function can be determined based on the inviscid, fully stalled and the steady lift curve:

$$f_s^{steady}(\alpha) = \frac{C_{l,Steady}(\alpha) - C_{l,Fs}(\alpha)}{C_{l,Inv}(\alpha) - C_{l,Fs}(\alpha)} \ .$$

- A typical inviscid lift curve could be the thin airfoil theory lift curve corrected for zero lift angle:

$$C_{l,Inv} = 2\pi(\alpha - \alpha_0) \ .$$

- Øye suggest to use half the slope of the steady curve for the fully stalled lift curve and merge with the steady curve around 30 degrees

Time delay effects

- To mimic the fact that the airfoil will not immediately adjust to the new conditions, we assume that the slope of the separation function can be computed from the following expression assuming a reasonable time constant:

$$\frac{df_s}{dt} = \frac{f_s^{steady} - f_s}{\tau} \ .$$

$$\frac{df_s}{dt} + \frac{1}{\tau} f_s = \frac{1}{\tau} f_s^{steady}$$

$$f_s^{dyn}(\Delta t) = f_s^{steady} + (f_s^t - f_s^{steady}) e^{\frac{-\Delta t}{\tau}} \ .$$

Input for the Øye model

- The Øye model needs the following input:
  - A static lift curve
  - A inviscid lift curve
  - A fully stalled lift curve
  - A value for the time constant τ, Øye suggest a value around:

$$\tau \sim 3\frac{c}{U_{eff}} \ .$$

The Øye model, practical implementation

At t=0 assume f_s to be equal to the steady value as initial condition.
For each time step:

1. Based on the instantaneous AoA determine the new steady value of f_s , ,using:

$$f_s^{steady}(\alpha) = \frac{C_{l,Steady}(\alpha) - C_{l,Fs}(\alpha)}{C_{l,Inv}(\alpha) - C_{l,Fs}(\alpha)} \ .$$

2. Compute the dynamic value of f_s ,, using the time increment:

$$f_s^{dyn}(t+\Delta t) = f_s^{steady} + [f_s^{dyn}(t) - f_s^{steady}]e^{\frac{-\Delta t}{\tau}} \ .$$

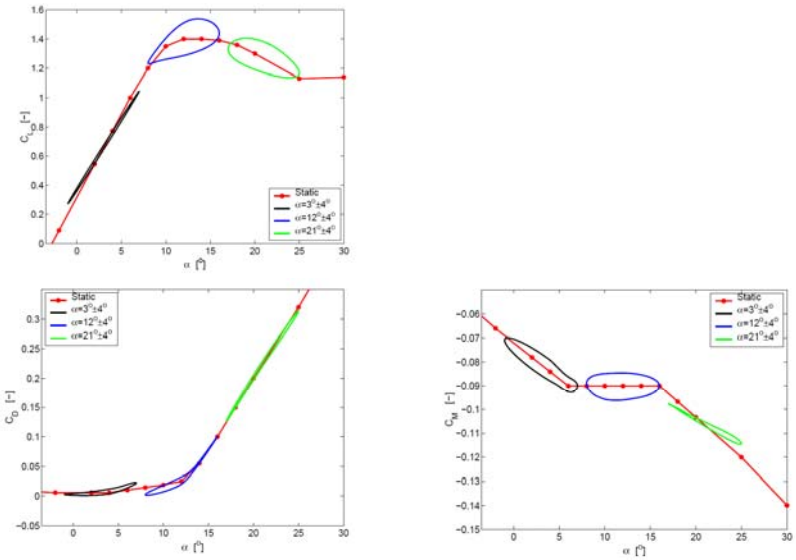
3. Compute the new instantaneous lift value using

$$C_{l,dyn}(\alpha) = f_s^{dyn}(\alpha)C_{l,Inv} + [1 - f_s^{dyn}(\alpha)]C_{l,Fs}(\alpha)$$

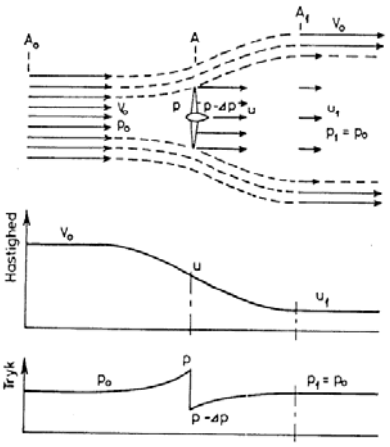
Dynamic stall modeling: Øye

- The dynamic stall model of Øye only models the unsteady lift.
- For typically wind turbine applications the unsteady viscous drag effects are typically small and are often neglected.
- In contrast to helicopter aerodynamics where the dynamic value of the pitching moment is very important, the high torsion stiffness of wind turbine blades makes this less important. In practice the unsteady pitching moment can often be neglected for typical load calculations.

Dynamic stall effects on lift, drag, moment



The energy conversion – Bernoulli



$$T = \Delta p A$$

Bernoulli

$$\tfrac{1}{2}\rho V^2 + p = H$$

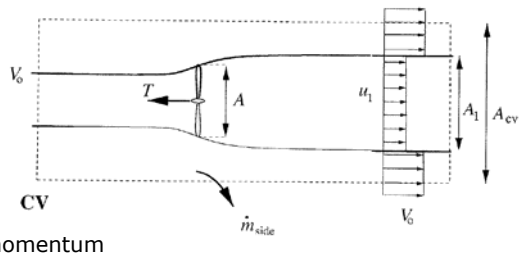
Applied before and after rotor disc

$$p_0 + \tfrac{1}{2}\rho V_0^2 = p + \tfrac{1}{2}\rho u^2$$

$$p - \Delta p + \tfrac{1}{2}\rho u^2 = p_0 + \tfrac{1}{2}\rho u_1^2$$

$$\Rightarrow \Delta p = \tfrac{1}{2}\rho\left(V_0^2 - u_1^2\right)$$

The energy conversion – 1D momentum



Change in momentum

$$\vec{T} = \dot{m}\Delta\vec{V} \qquad \Rightarrow T = \rho V_0 A_{CV} V_0 - \rho u_1 A_1 u_1 - \rho V_0 (A_{CV} - A_1) V_0 - \dot{m}_{side} V_0$$

Conservation of mass and insert in momentum equation

$$\rho A_{CV} V_0 = \rho A_1 u_1 + \rho (A_{CV} - A_1) V_0 + \dot{m}_{side} \qquad \Rightarrow \dot{m}_{side} = \rho A_1 (V_0 - u_1)$$

$\dot{m} = \rho u A = \rho u_1 A_1$ yields $T = \dot{m}(V_0 - u_1)$

Combining momentum and Bernoulli

$$u = \tfrac{1}{2}(V_0 - u_1)$$

1D momentum continued

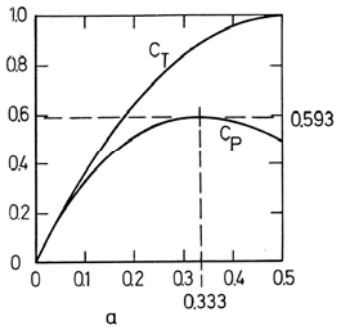
Introducing the induction factor a

$$u = (1-a)V_0 \qquad u_1 = (1-2a)V_0$$

Extracted power for an ideal rotor is difference in power from inlet to outlet

$$P = \tfrac{1}{2}\dot{m}\left(V_0^2 - u_1^2\right)$$

$$C_p = \frac{P}{P_{avail}} = \frac{2\rho A V_0^3 a(1-a)^2}{\frac{1}{2}\rho V_0^3 A} = 4a(1-a)^2$$

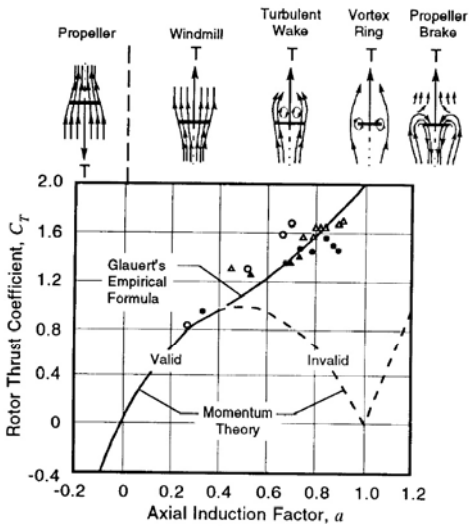


Similar for the torque

$$T = \rho u A(V_0 - u_1)$$

$$C_T = \frac{T}{T_{avail}} = \frac{2\rho A V_0^2 a(1-a)}{\frac{1}{2}\rho V_0^2 A} = 4a(1-a)$$

The relation between thrust and axial induction



$$u = (1-a)V_0$$

$$C_T = 4a(1-a)$$

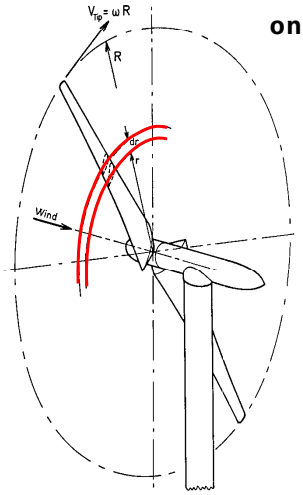
$$C_T = \frac{T}{\frac{1}{2}\rho V_0^2 A}$$

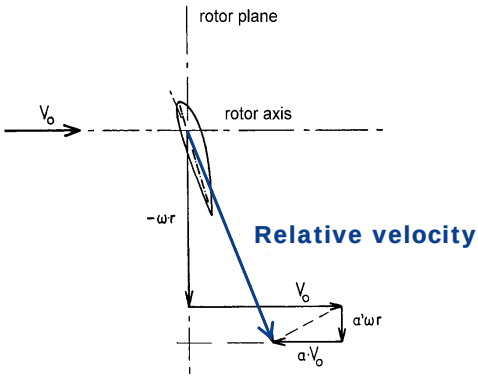
Blade Element Momentum theory

Rotor plane is discretized into

independent concentric annular elements

on which the 1D assumption is applied





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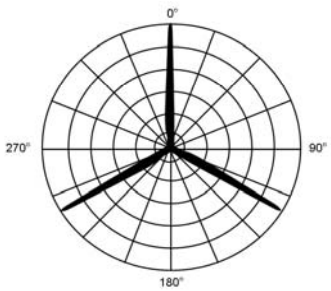
Induction model in HAWC2



A non-rotating grid is defined in which induced velocities are calculated. Enables e.g. possibility for different induction in top-bottom.

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BEM in HAWC2 formulated locally in grid.

$$\textit{Blade element theory}: \quad dT = \frac{1}{2}\rho W^2 C_y(\alpha) c N_B$$

Axial thrust at given radial station on the blade

$$CT = \frac{dT}{\frac{1}{2}\rho V_\infty^2 2\pi R} \Rightarrow$$

The thrust coefficient is found

$$CT = \frac{W^2 C_y(\alpha) c N_B}{2\pi r V_\infty^2}$$

V_∞ is the magnitude of the local free wind speed.

$$a \equiv -\frac{u_{induc,axial,local}}{\left|V_{\infty,local}\right|}$$

BEM in HAWC2 continued

$$\textit{Momentum theory}: \quad CT = 4a(1-a)$$

for $a > 0.33$ the Glauert empirical correlation between a and CT is used

The solution to the above equation is written on the form:

$$a = k_3 CT^3 + k_2 CT^2 + k_1 CT + k_0$$

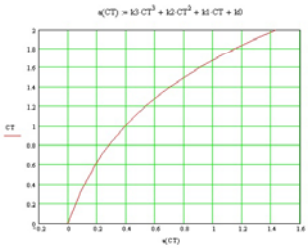
However, the $a = f(CT)$ relation can alternatively be derived from an actuator disc computation on the rotor and would then be a function of position on the rotor disc

k(3)=0.0892074

k(2)=0.0544955

k(1)=0.251163

k(0)=-0.0017077



Tangential wake rotation

Angular momentum on annular ring

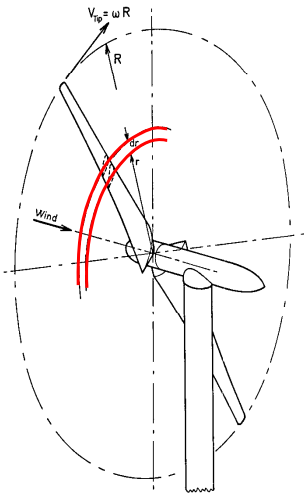
$$dQ = \rho(2\pi\, r\, dr)(1-a)V_\infty(2a'\Omega)$$

Tangential blade forces on annular ring

$$dQ = \tfrac{1}{2}\rho W^2 C_x(\alpha)cN_B r\, dr$$

Above equations solved for the tangential induction factor

$$a' = \frac{W^2 C_x(\alpha)cN_B}{8\pi r^2(1-a)V_\infty\Omega}$$



Tip correction

- Tip-loss
  - Inherent in BEM method: Forces smeared out. Corresponds to an infinite number of blades.
  - Prandtl suggested we can define a tip loss factor *F* which is the ratio between the load distribution for an optimally loaded rotor and the load distribution for a rotor with an infinite number of blades that gives the same induced velocity.

$$F = \frac{\frac{dT}{dr}}{\frac{dT}{dr}_\infty} \Leftrightarrow \frac{dT}{dr} = F\frac{dT}{dr}_\infty$$

  - The derivation of the correction is very complex, and is based on the vortex system behind a rotor with a finite number of blades. Original solution by Prandtl.

$$F = \frac{2}{\pi}\cos^{-1}\left(\exp\left(-\frac{N_B}{2}\frac{R-r}{r\sin\phi}\right)\right)$$

Tip loss correction in HAWC2

- The tiploss model used in HAWC2 is based on the modified expression by Wilson and Lissaman

$$F = \frac{2}{\pi}\cos^{-1}\left(\exp\left(-\frac{N_B}{2}\frac{(1-r/R)}{(r/R)\sin\phi}\right)\right)$$

- This factor is inserted in the induction calculation

$$CT = 4a\ F(1-a) \quad or \quad \frac{CT}{F} = 4a(1-a)$$

$$CT^* = \frac{CT}{F}$$

$$a = k_3\, CT^{*3} + k_2\, CT^{*2} + k_1\, CT^*$$

BEM procedure

- For each point in the polar grid
  1. Get the local wsp and use local induced wsp from previous timestep/iteration
  2. Get the local CT calculated at the two neighboring blades if the local wsp and induction was used there.
  3. Account for Prandtl tiploss
  4. Interpolate CT based on azimuthal distance between grid point and the two blades. Now the local CT is known.
  5. Calculate local induction factor a
  6. Correct mean level of a based on skew inflow angle.
  7. Calculate tangential induction factor a_m
  8. Calculate induced wind speeds axial (1) and tangential
  9. Correct for azimuthal variations of axial induction u related to skew inflow angle incl. axial induction influence..
  10. Update u and u_t in time based on two first order low pass filters. One for nearwake contribution and one for farwake.

- For each point on the blade
  11. Get the induced velocity based on azimuthal interpolation of the two closest grid points at same radius.

Corrections for yawed inflow

General case of momentum for rotor, Glauert

$$\vec{T} = \dot{m}\Delta\vec{V} = A\rho\left|\vec{V}_\infty + \vec{u}_i\right|\left(\vec{V}_\infty - \left(\vec{V}_\infty + 2\vec{u}_i\right)\right) \Rightarrow \vec{T} = A\rho\left|\vec{V}_\infty + \vec{u}_i\right|\left(-2\vec{u}_i\right)$$

Which for axial flow is

$$T_z = A\rho\left|\vec{V}_\infty + \vec{u}_i\right|\left(-2u_{i,z}\right) \Rightarrow T_z = A\rho\left|u_d\right|\left(-2u_{i,z}\right)$$

Geometry relation

$$\left|u_d\right|^2 = \left(\left|V_\infty\right|\cos\Phi + u_{i,z}\right)^2 + \left(\left|V_\infty\right|\sin\Phi\right)^2$$

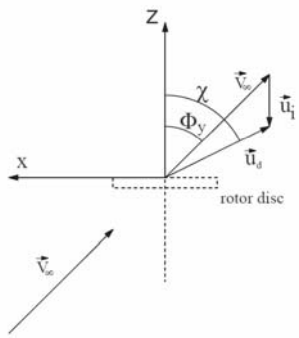
$$\Rightarrow \left|u_d\right|^2 = \left|V_\infty\right|^2\left(1 + \frac{u_{i,z}^2}{\left|V_\infty\right|^2} + \frac{2\cos\Phi\, u_{i,z}}{\left|V_\infty\right|}\right)$$

$$a \equiv -\frac{u_{i,z}}{\left|V_{\infty,local}\right|}$$

$$T_z = \rho A\sqrt{\left|V_\infty\right|^2\left(1 + \frac{u_{i,z}^2}{\left|V_\infty\right|^2} + \frac{2\cos\Phi\, u_{i,z}}{\left|V_\infty\right|}\right)}\left(-2u_{i,z}\right)$$

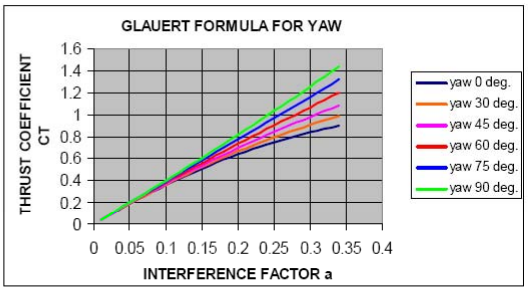
$$T = \tfrac{1}{2}A\rho V_0^2\, 4a\left(1 + a^2 - 2a\cos\Phi\right)^{1/2}$$

$$CT = \frac{T}{\tfrac{1}{2}A\rho V_0^2} = 4a\left(1 + a^2 - 2a\cos\Phi_y\right)^{1/2}$$



Yawed inflow

$$CT = 4a\left(1 + a^2 - 2a\cos\Phi_y\right)^{1/2}$$



Azimuthal impact on yawed inflow

$$u_{ix} = u_i\left(1 + k_x\, r\cos(\psi) + k_y\, r\sin(\psi)\right)$$

Ψ Is the azimuth angle

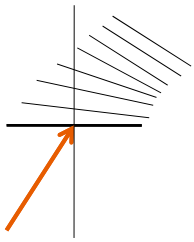
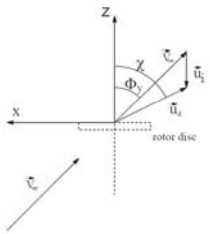


Table 3.1. *Various Estimated Values of First Harmonic Inflow*

Author(s)	k_x	k_y
Coleman et al. (1945)	$\tan(\chi/2)$	0
Drees (1949)	$(4/3)(1-\cos\chi-1.8\mu^2)/\sin\chi$	-2μ
Payne (1959)	$(4/3)(\mu/\lambda/(1.2+\mu/\lambda))$	0
White & Blake (1979)	$\sqrt{2}\sin\chi$	0
Pitt & Peters (1981)	$(15\pi/23)\tan(\chi/2)$	0
Howlett (1981)	$\sin^2\chi$	0



In HAWC2 the Coleman method is used with a constant of 0.4 instead of 0.5

$$\tan(\chi) = \frac{V_\infty \sin(\Phi_y)}{V_\infty \cos(\Phi_y - u_i)}$$

Yawed inflow

The reduction K_a of the induction factor a as function of yaw angle and thrust coefficient wass shown in the figure above and in order to use the correction directly in the induction calculation the following polynomial fit has been derived:

$$K_a = CT^3 k_3 + CT^2 k_2 + CT\, k_1$$

where the parameters k_3, k_2 and k_1 depends on the yaw angle (radians) in the following way:

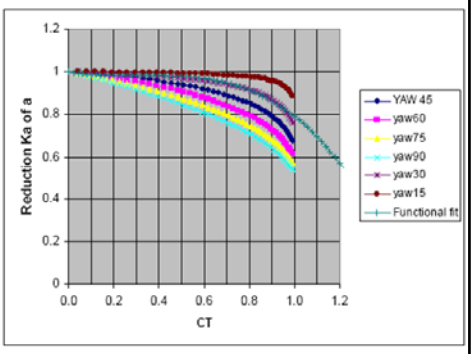
$$k_3 = -0.6481\Phi_y^{\,3} + 2.1667\Phi_y^{\,2} - 2.0705\Phi_y$$

$$k_2 = 0.8646\Phi_y^{\,3} - 2.6145\Phi_y^{\,2} + 2.1735\Phi_y$$

$$k_1 = -0.164\Phi_y^{\,3} + 0.4438\Phi_y^{\,2} - 0.5136\Phi_y$$

Modification of induction

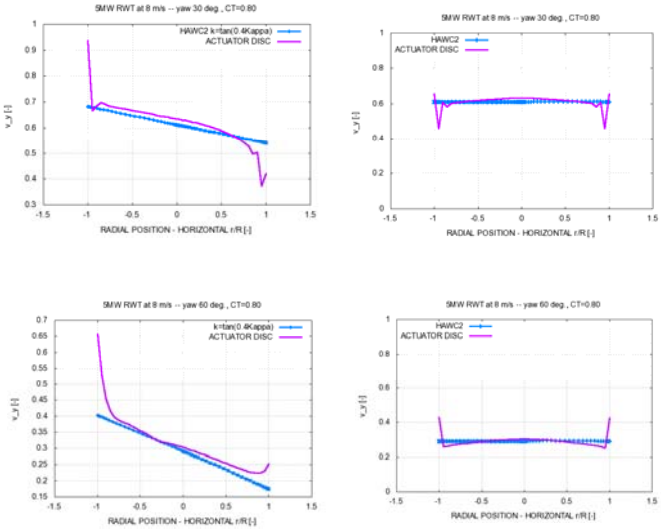
$$a_{cor} = K_a a$$



Comparison between HAWC2 and FIDAP

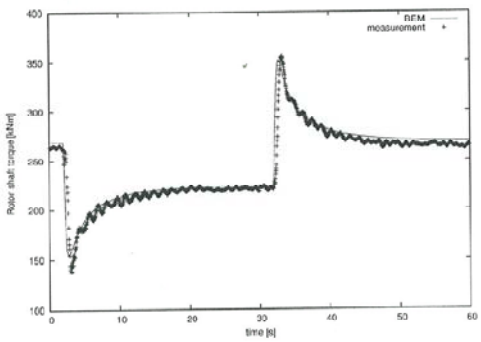
30deg

60deg



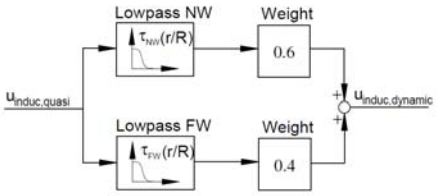
Effect of dynamic inflow

- Dynamic wake:
  - Taking into account, that there is a build-up time for the wake of the turbine, and thus the loading, after a sudden change in inflow.
  - This may be modeled as a filter, for instance consisting of two first order differential equations



Dynamic inflow

The Dynamic indflow ins handled by two first order filters coupled in parallel with weight factors



$$\tau_{FW}\left(r/R\right)=\tau^{*}_{FW}\left(r/R\right)\frac{R}{V_{\infty,y}\,\mathrm{MAX}\left[1+3\frac{V_{induc,y}}{V_{\infty,y}},0.2\right]}$$

$$\tau_{NW}\left(r/R\right)=\tau^{*}_{NW}\left(r/R\right)\frac{1.8R}{V_{\infty,y}\,\mathrm{MIN}\left[1-3\frac{V_{induc,y}}{V_{\infty,y}},2.0\right]}$$

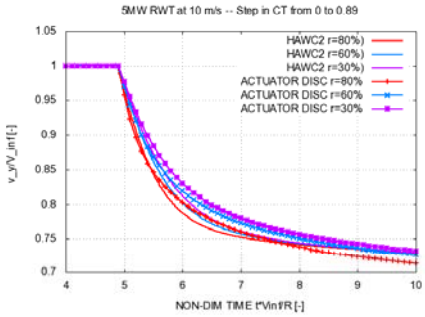
The non-dim time constants was found to approximately 2.0 for the far wake contribution and 0.5 for the near wake part

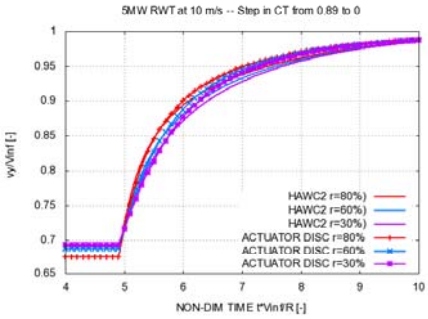
Dynamic inflow

$$\tau^{*}_{NW}\left(r/R\right)=k_2\left(r/R\right)^2+k_1\left(r/R\right)^1+k_0$$

	NW	*FW*
k2	-0.4783	-0.4751
k1	0.1025	0.4101
k0	0.6125	1.9210

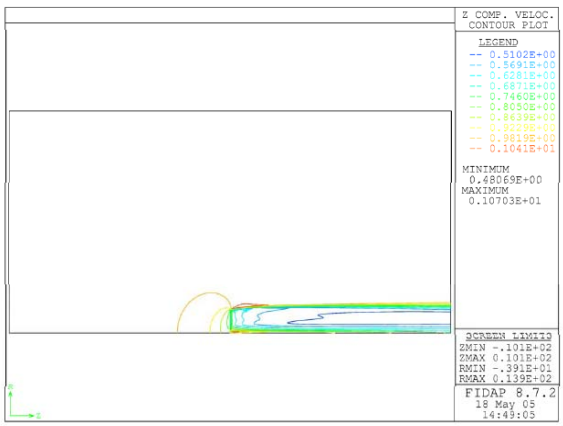
Dynamic inflow, comparison between HAWC2 and FIDAP



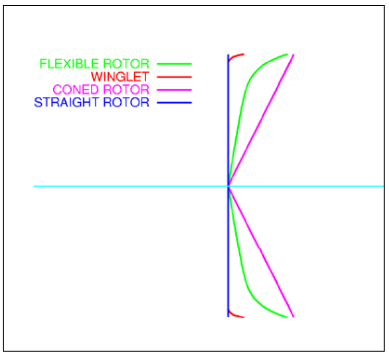


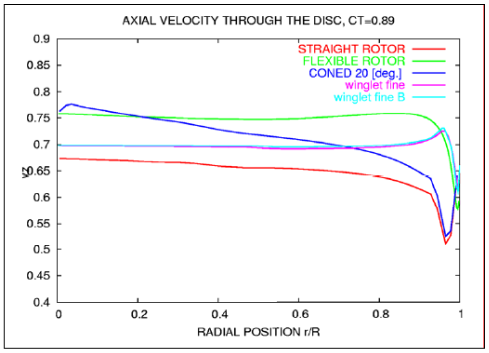
Actuator disc computations

Axial velocity contours



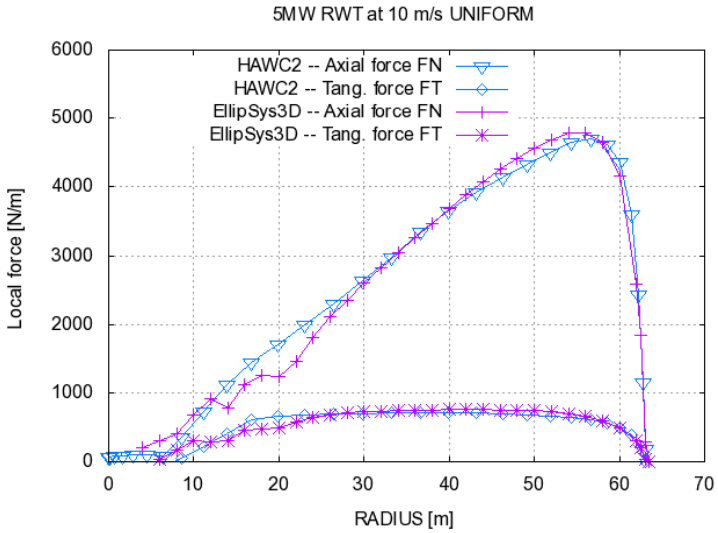
Examples of limitations of the BEM model



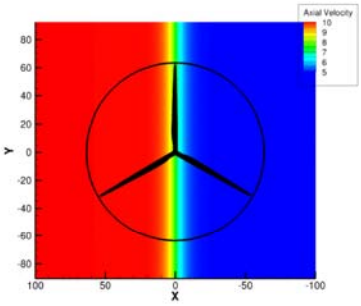


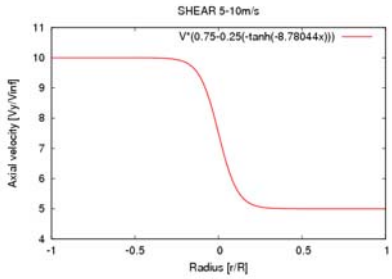
For such special rotors a user defined a-ct relation can be used if the characteristic is known from e.g. CFD.

Comparison

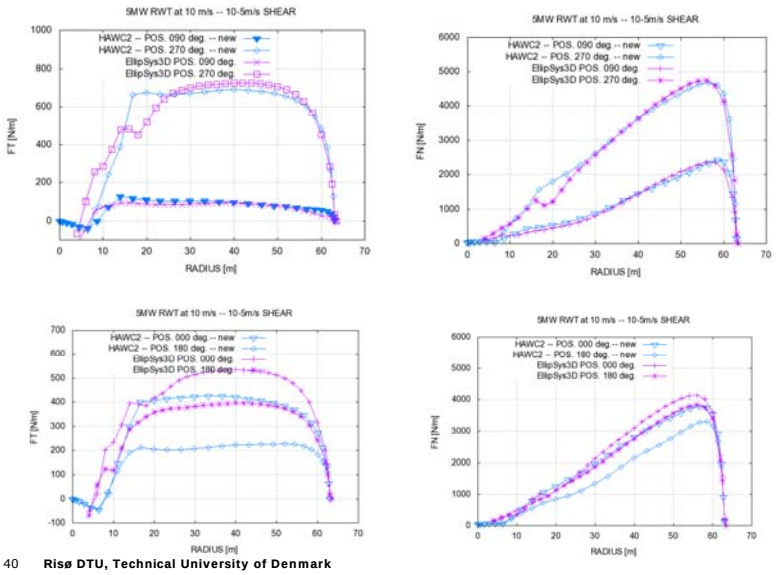


Special shear (1)

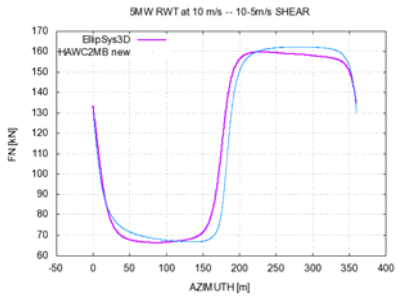


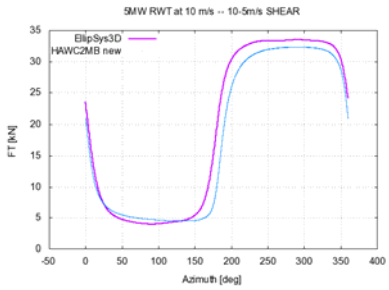


Special shear (2)



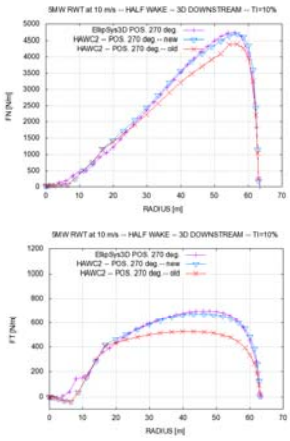
Special shear (3)

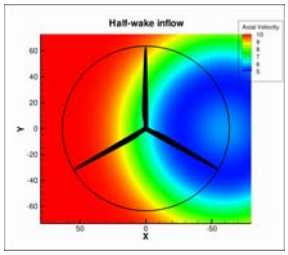


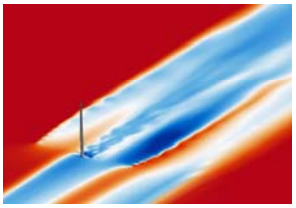


Wake situation

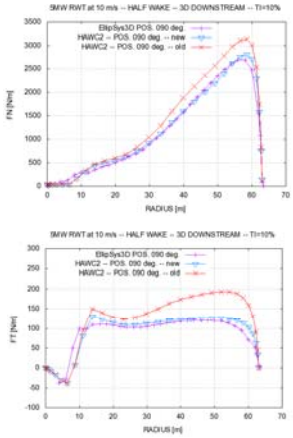
270°





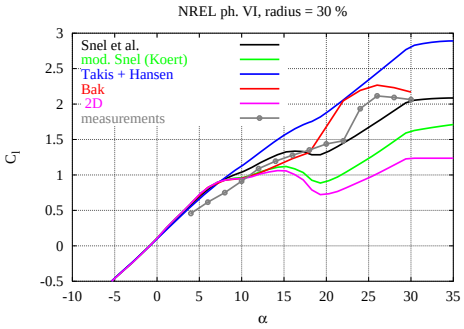


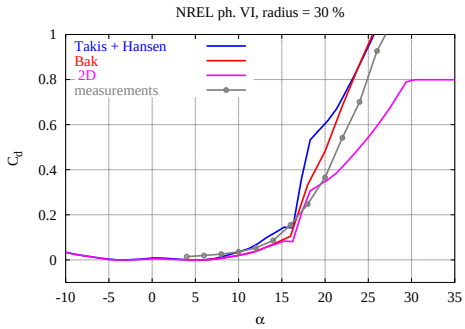
90°



3D effects on profile coefficients

NREL-test: Airfoil data, r/R=30%





More on profile coefficients

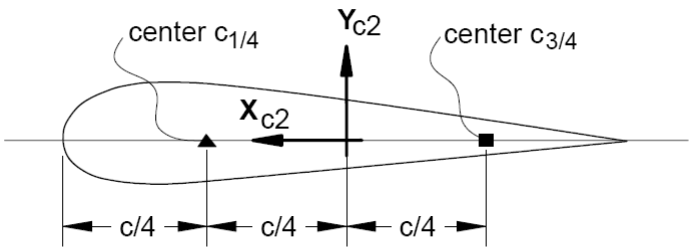
- Aerodynamic coefficients usually obtained from 2D wind tunnel tests, limited angle of attack range
- Extend airfoil data over a larger angle of attack range
- Apply 3D corrections
- Some useful references:
  - B. Montgomerie, Methods for root effects, tip effects and extending the angle of attack range to +-180 deg, with application to aerodynamics for blades on wind turbines and propellers. FOI - Swedish Defence Research Agency, 2004, URL: http://www2.foi.se/rapp/foir1305.pdf
  - C. Lindenburg, Investigation into Rotor Blade Aerodynamics. 2003, URL: www.ecn.nl/publications/PdfFetch.aspx?nr=ECN-C--03-025
  - S.-P. Breton, F. N. Coton, and G. Moe, "A study on rotational effects and different stall delay models using a prescribed wake vortex scheme and NREL phase VI experiment data," Wind Energy, vol. 11, no. 5, pp. 459-482, 2008, doi:10.1002/we.269

Aerodynamic input in HAWC2

```
begin aero ;
  nblades  3;
  hub_vec hub_center 3 ;  rotor rotation vector (normally shaft composant
  ;                       directed from pressure to sustion side)
  link 1 mbdy_c2_def blade1;
  link 2 mbdy_c2_def blade2;
  link 3 mbdy_c2_def blade3;
  ae_filename        ./data/H2_blade_ae_e112.dat ;
  pc_filename        ./data/H2_blade_pc_e112.dat ;
  induction_method   1 ;     0=none, 1=normal
  aerocalc_method    1 ;     0=no aerodynamic, 1=with aerodynamic
  aerosections       30 ;
  ae_sets            1 1 1;
  tiploss_method     1 ;     0=none, 1=prandtl
  dynstall_method    2 ;     0=none, 1=stig øye method,2=mhh method
end aero ;
```


Aerodynamic centers

Position of aerodynamic centers related to c2_def section coo.



Aerodynamic layout

```
1 Aerodynamic planform                    Nset
1 22                                      Setnr    Ndata
  0       3.5      100      1             radius   chord    thick pc_set
  0.5     3.5      100      1
  2.1     3.5      99.9734  1
  4.9     4.2      74.46957 1
  7.7     4.9      48.96571 1
  10.5    5.6      35.71321 1
  13.3    5.75     28.31165 1
  16.1    5.656    23.86846 1
  18.9    5.2787   21.97113 1
  21.7    4.9014   20.58616 1
  24.5    4.5241   19.67198 1
  27.3    4.14736  19.04802 1
  30.1    3.84814  18.19035 1
  32.9    3.5489   17.32931 1
  35.7    3.24968  16.76627 1
  38.5    2.95046  16.33576 1
  41.3    2.65123  15.807   1
  44.1    2.352    15.32908 1
  47.033  1.83467  15.65677 1
  49.967  1.31733  15.41907 1
  52.26   0.913    14.99    1
  52.9    0.8      14.7875  1
```


Profile coefficients file

```
1 Converted       from     Bladed   format
8 nsets
1 65     14.5                        P145_2_a
  -180   0        0.018    0.041
  -170   0.52     0.1121   0.3075
  -160   0.8      0.3271   0.328
  -150   0.658    0.5209   0.3485
  -140   0.58     0.7119   0.353
  -130   0.501    0.9115   0.3475
  -120   0.401    1.0929   0.342
  -110   0.279    1.2316   0.3376
  -100   0.141    1.3074   0.3225
  -90    0        1.308    0.303
  -80    -0.176   1.3074   0.29
  -70    -0.348   1.2316   0.275
  -60    -0.501   1.0929   0.2475
  -50    -0.626   0.9115   0.22
  -40    -0.725   0.7119   0.192
  -30    -0.822   0.5209   0.151
  -20    -0.931   0.3424   0.0406
```

Aerodynamic forces on other components

- A simple drag model can be linked to other main_bodies.

```
begin aerodrag ;
  begin aerodrag_element ;
    mbdy_name tower;
    aerodrag_sections uniform 10 ;
    nsec 2 ;
    sec 0.0 0.6 12.0 ;  tower bottom
    sec 100.0 0.6 4.0 ;   tower top
  end aerodrag_element;
;
 begin aerodrag_element ;
    mbdy_name nacelle;
    aerodrag_sections uniform 2 ;
    nsec 2 ;
    sec 0.0 0.6 4.0 ;
    sec 5.0 0.6 4.0 ;
  end aerodrag_element;
end aerodrag;
```

NB! Physical length in meters!
Error in manual here!

Exercise

- Perform simulation for step changes in wind, constant rpm and pitch, wind speeds ranging from 4-12m/s. Major outcome is to check that the input data are OK.