

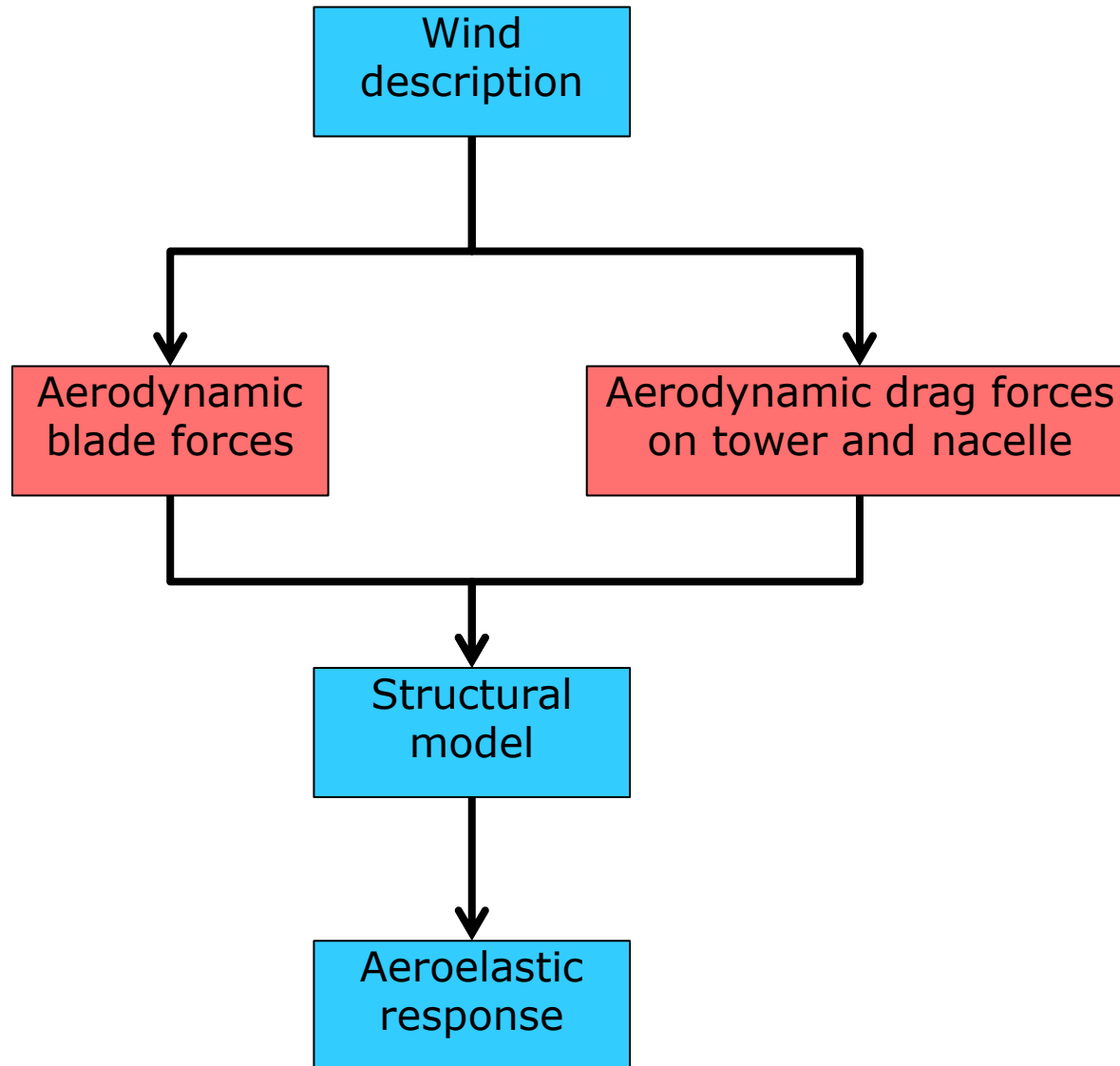
HAWC2 course

Lesson 3: Aerodynamic modeling and implementation in HAWC2

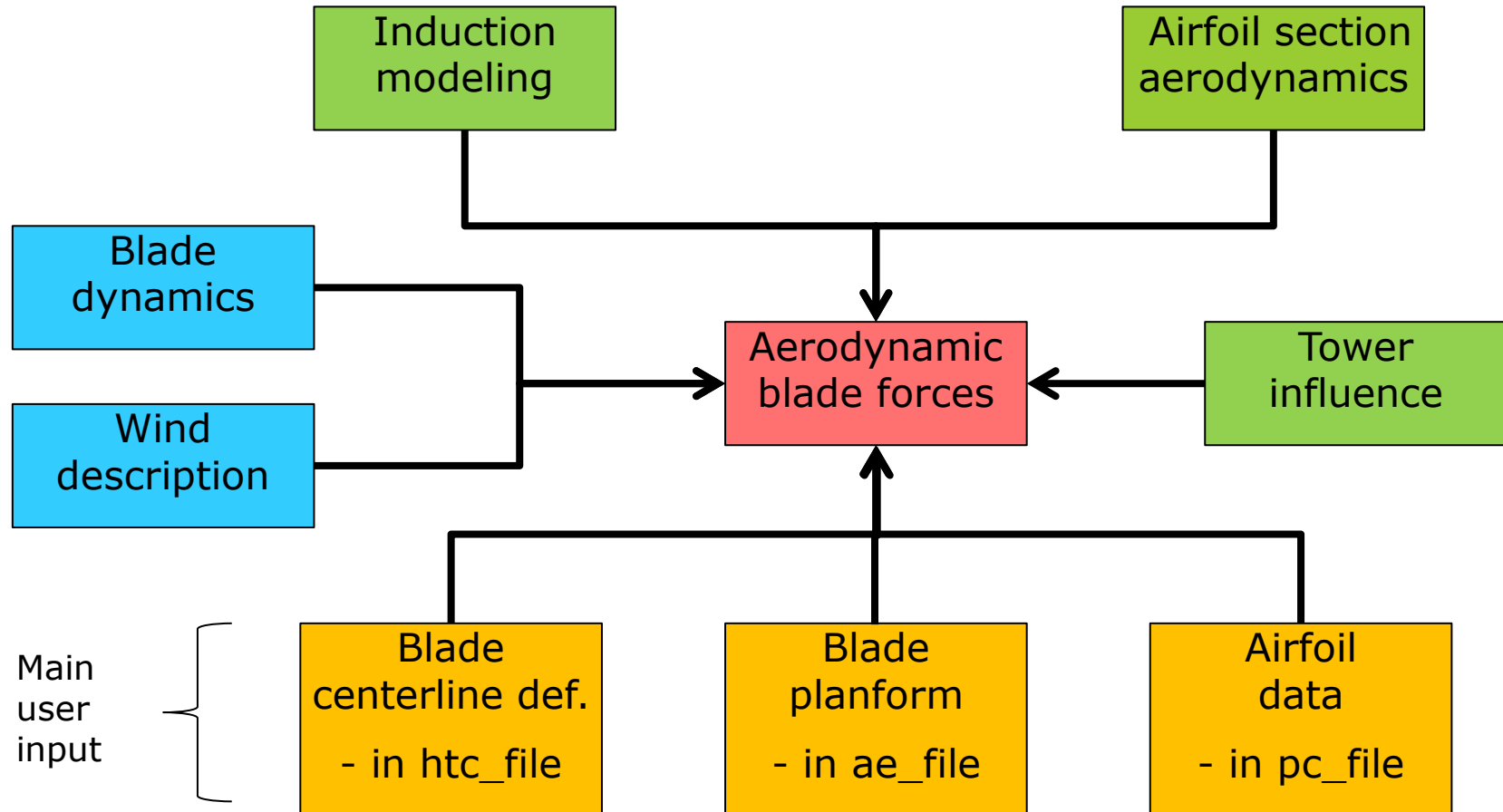
$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$

$$\Delta \int_a^b \varepsilon \Theta^{\sqrt{17}} + \Omega \int \delta e^{i\pi} = \{2.7182818284\}$$

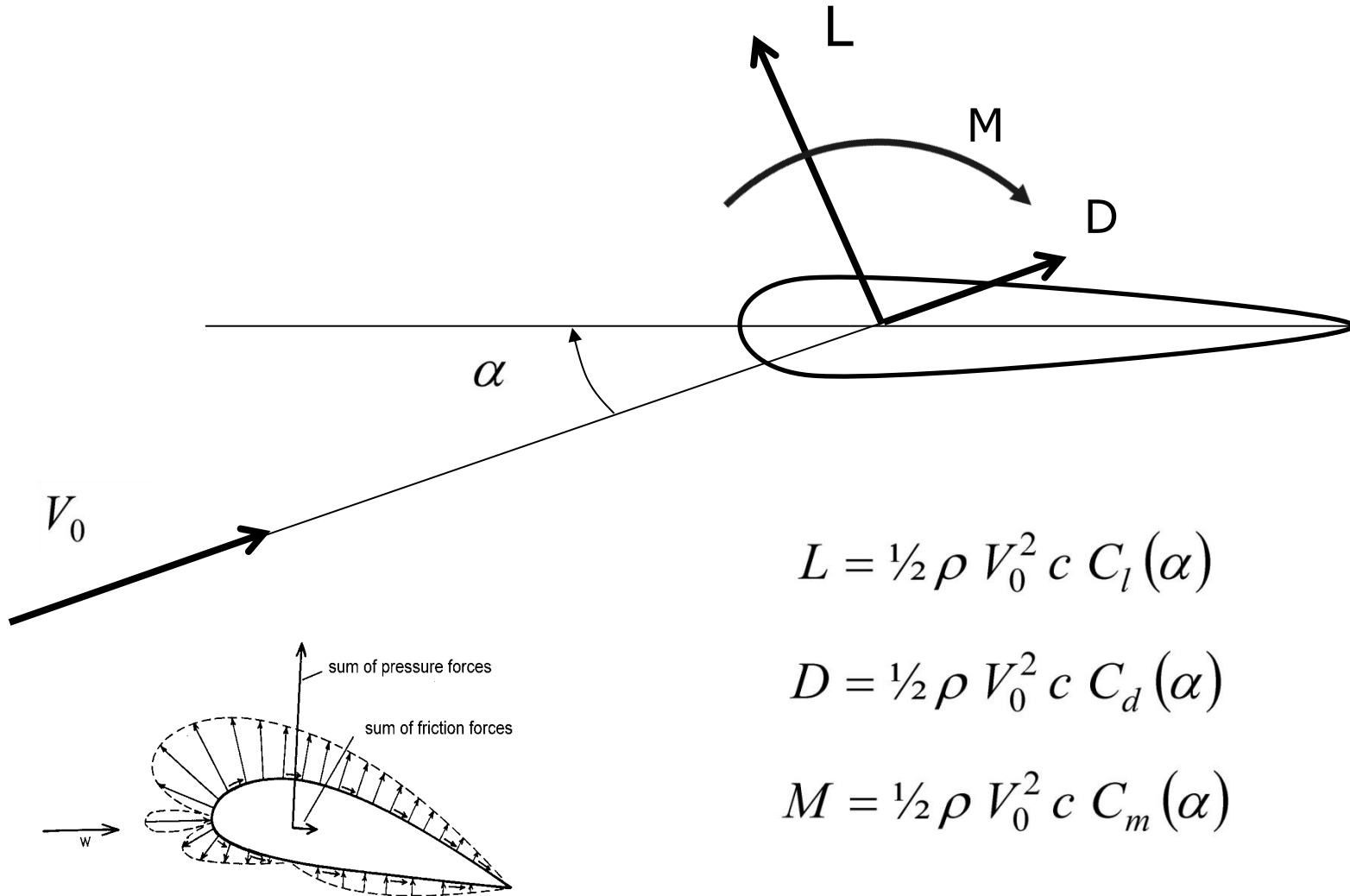
$$\infty = \chi^2 \sum! >$$



Overview of aerodynamic force calculation



Airfoil section aerodynamics



$$L = \frac{1}{2} \rho V_0^2 c C_l(\alpha)$$

$$D = \frac{1}{2} \rho V_0^2 c C_d(\alpha)$$

$$M = \frac{1}{2} \rho V_0^2 c C_m(\alpha)$$

Stall types

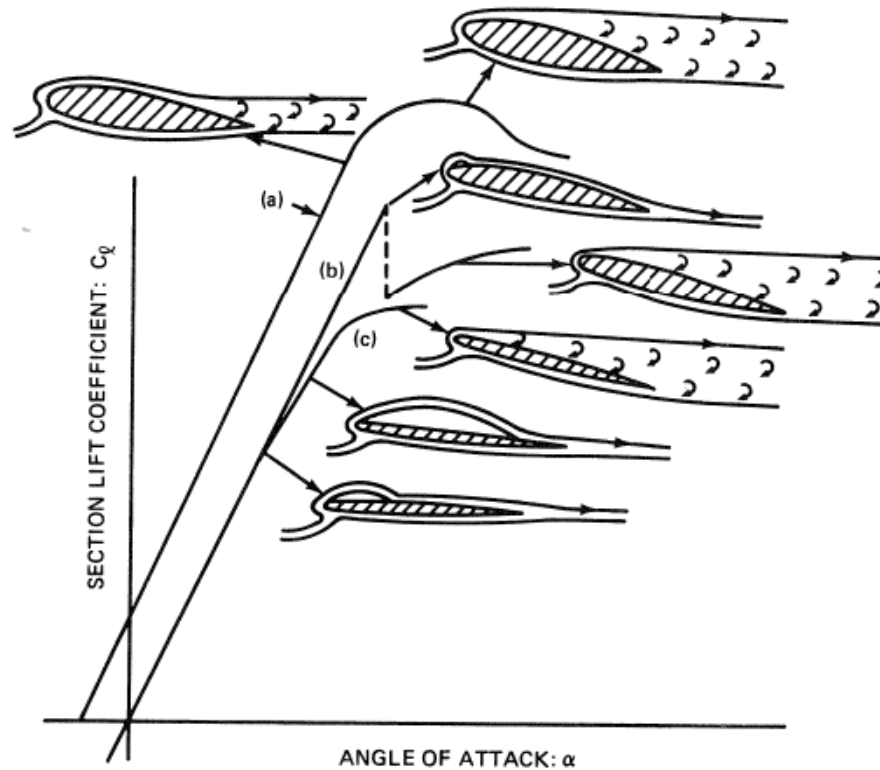
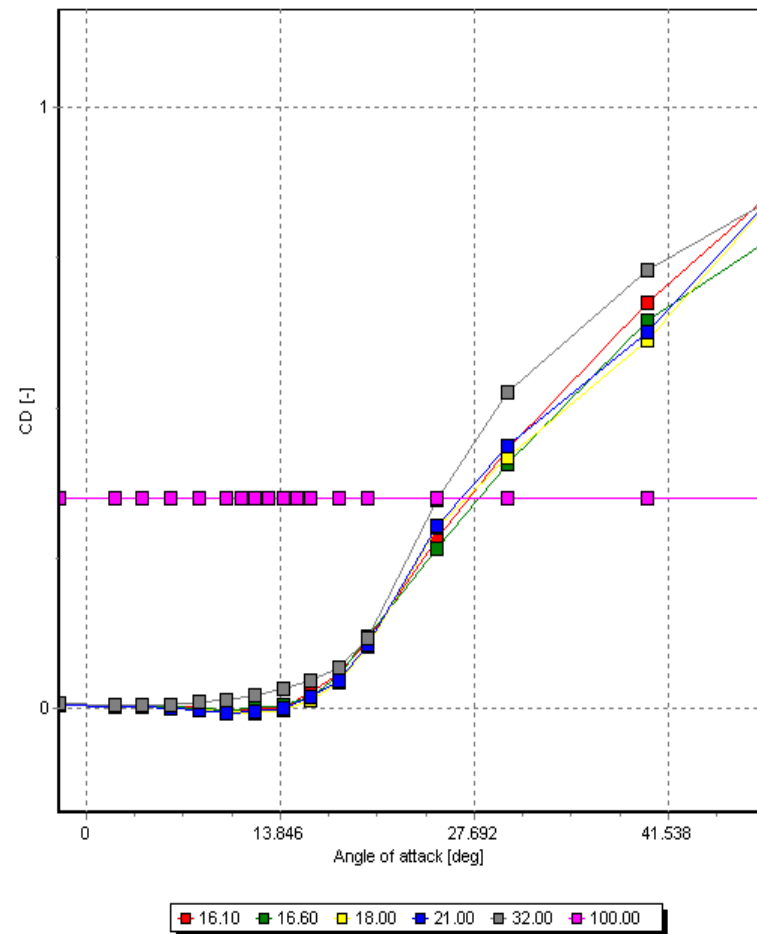
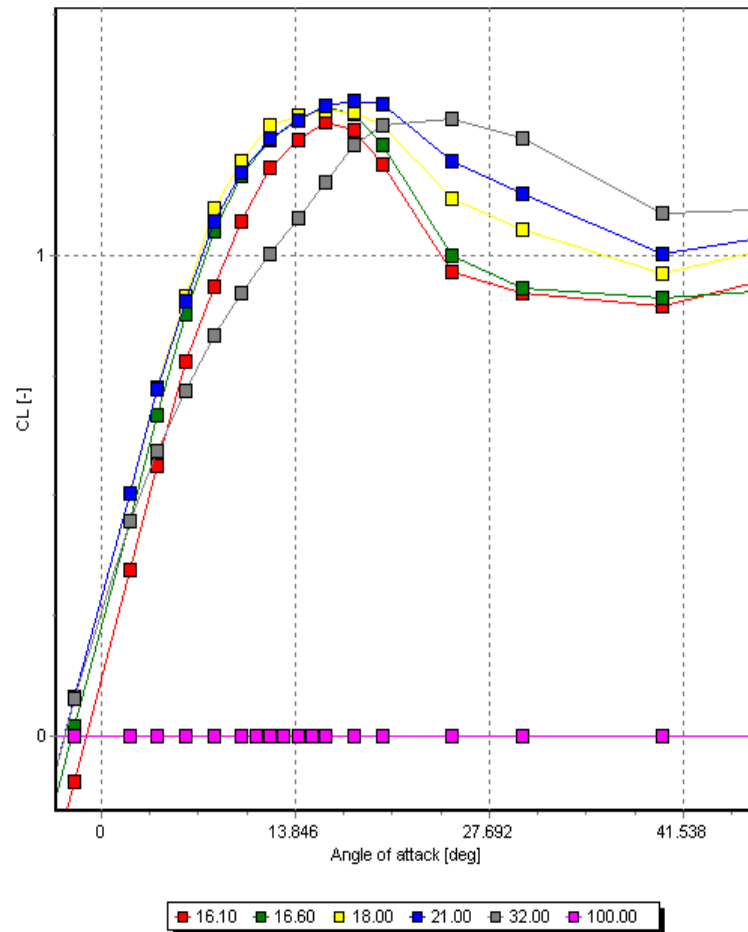


Figure 6.18 Three types of stall: (a) trailing-edge, (b) leading-edge, and (c) thin-airfoil

Lift and drag – example

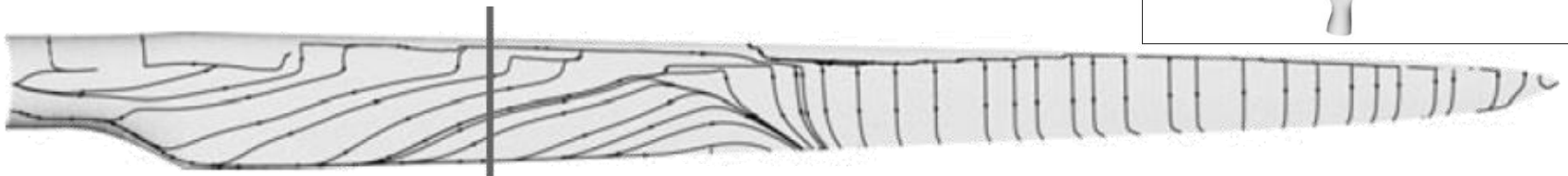
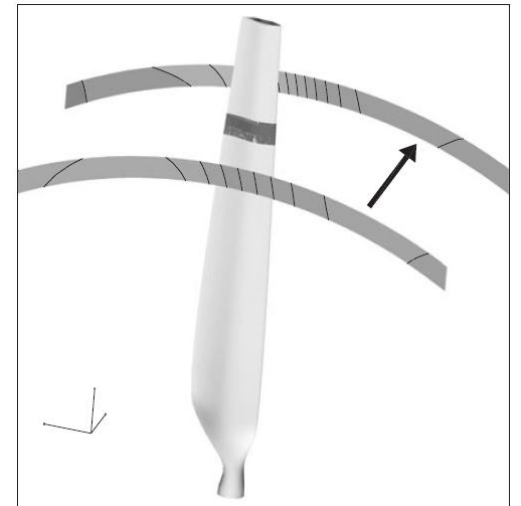
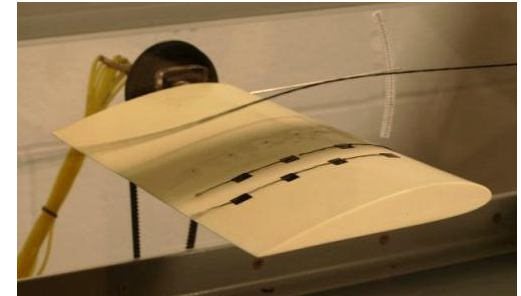


$$L = \frac{1}{2} \rho W^2 c C_L(\alpha)$$

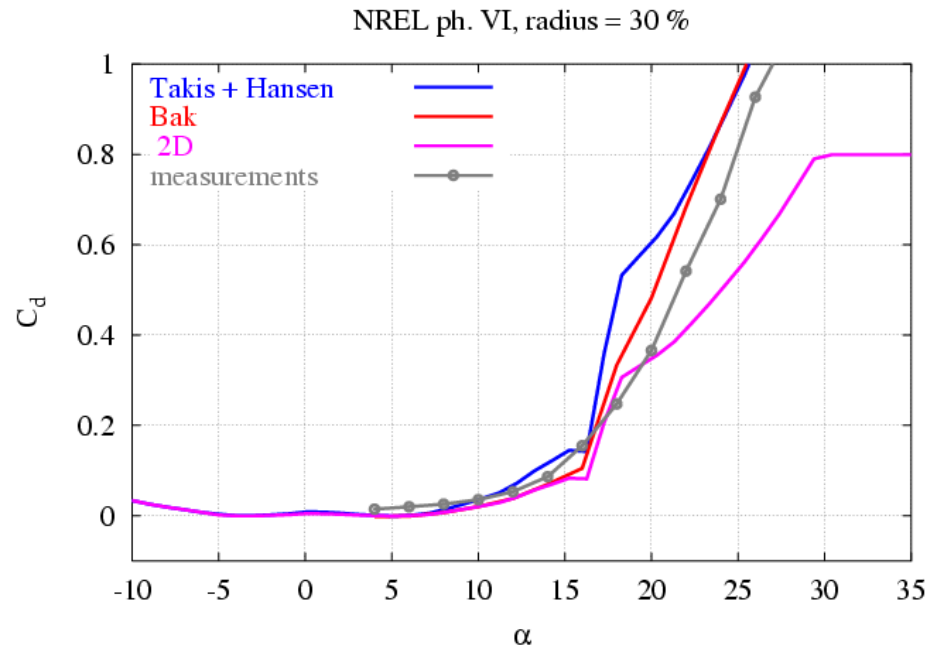
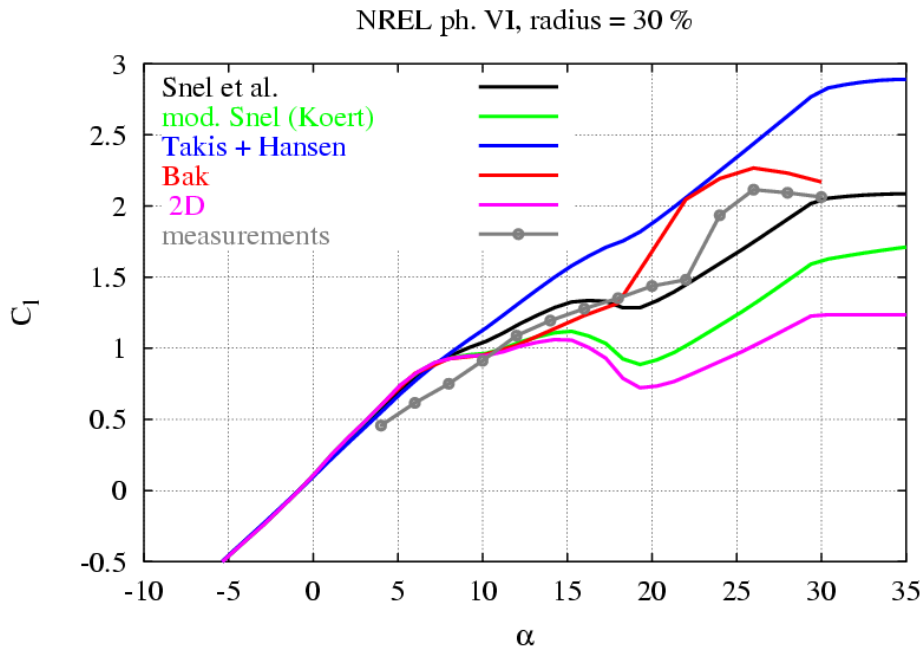
$$D = \frac{1}{2} \rho W^2 c C_D(\alpha)$$

Profile coefficients: how to retrieve them

- Where to get the profile coefficients from:
 - 2D simulations (panel codes, CFD)
 - 2D wind tunnel tests
- Limited angle of attack range:
 - Extend airfoil data over a larger angle of attack range
- 2D simulation/measurements:
 - Apply 3D corrections: stall delay effects



Profile coefficients: 3D corrections



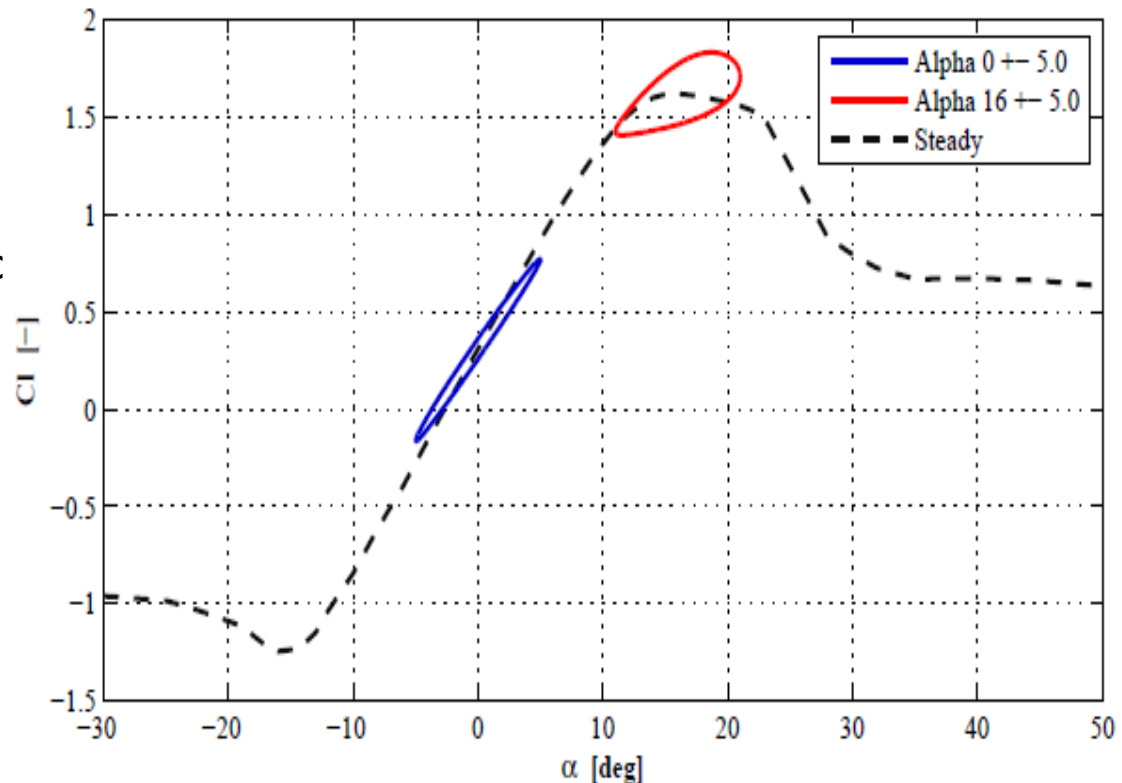
Some useful references:

- B. Montgomerie, *Methods for root effects, tip effects and extending the angle of attack range to ± 180 deg, with application to aerodynamics for blades on wind turbines and propellers*. FOI - Swedish Defence Research Agency, 2004, URL: <http://www2.foi.se/rapp/foir1305.pdf>
- C. Lindenburg, *Investigation into Rotor Blade Aerodynamics*. 2003, URL: www.ecn.nl/publications/PdfFetch.aspx?nr=ECN-C--03-025
- S.-P. Breton, F. N. Coton, and G. Moe, "A study on rotational effects and different stall delay models using a prescribed wake vortex scheme and NREL phase VI experiment data," *Wind Energy*, vol. 11, no. 5, pp. 459-482, 2008, doi:10.1002/we.269

Life is dynamic...

2D aerodynamic model should account for dynamics (unsteady forces) in:

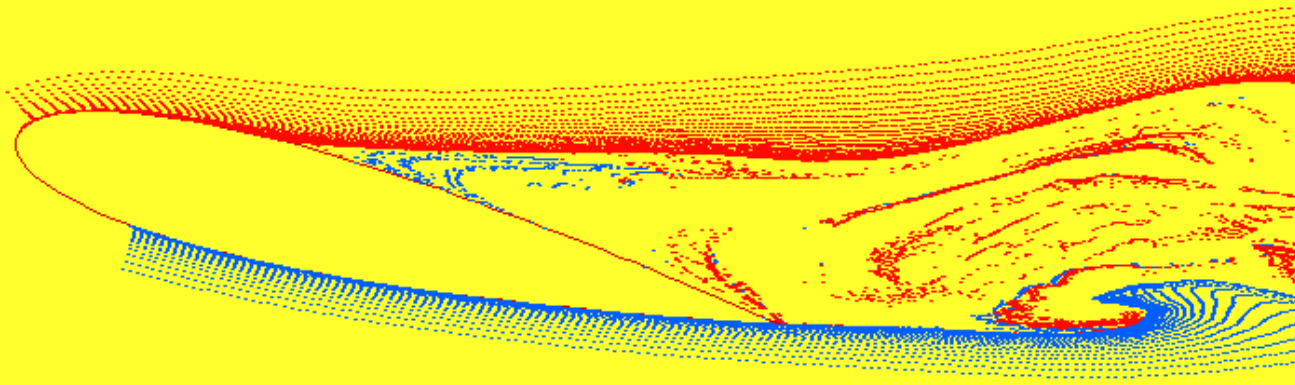
- **Attached flow** :
Memory effects of vorticity shed into the wake.
- **Flow separation** (dynamic stall):
Dynamics of the boundary layer, delay in flow separation.
- **Non-circulatory terms**:
added mass, acceleration terms.



Results from CFD: Dynamic stall

EllipSys2D, Dynamic Stall
Afd. for Vindenergi
Risø

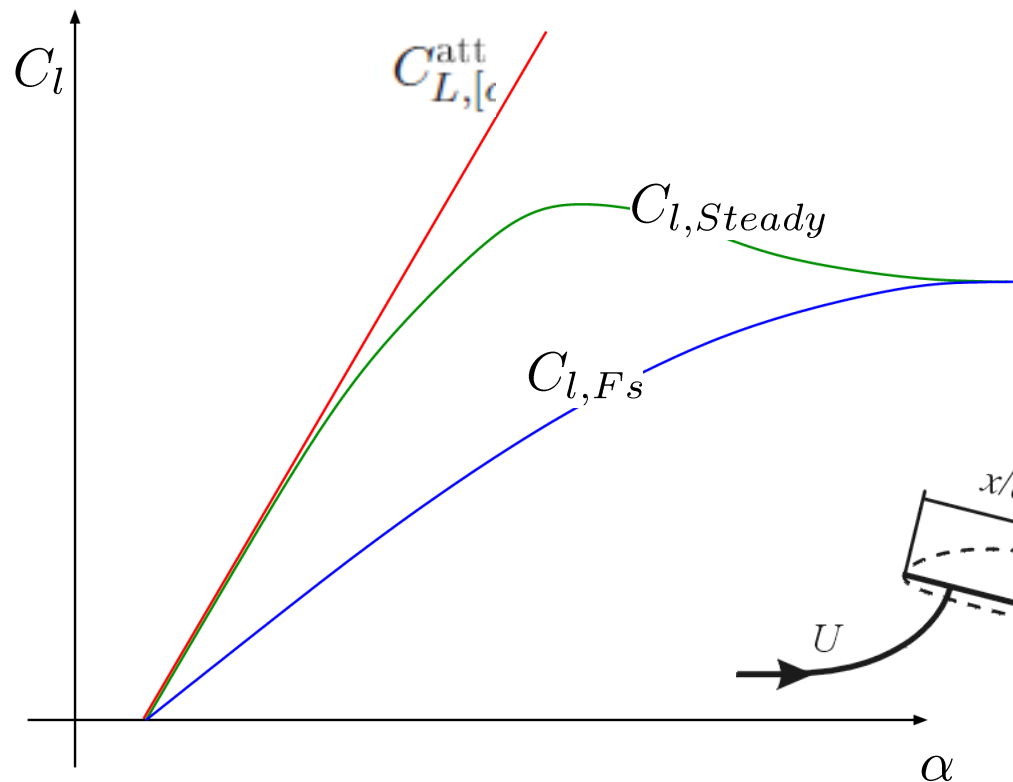
AOA= 12.90



Dynamic stall model (1)

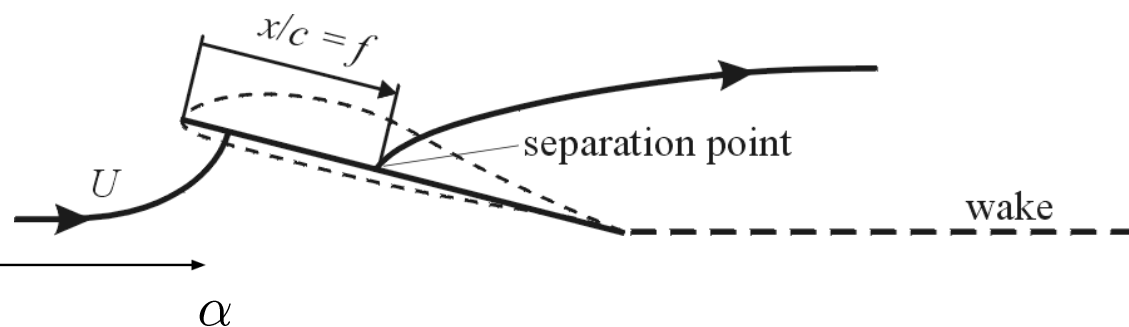
- Lift as weighted sum:

$$C_L^{\text{Circ.Dyn}} = C_{L,[\alpha_{\text{eff}};\beta_{\text{eff}}]}^{\text{att}} f^{\text{dyn}} + C_{L,[\alpha_{\text{eff}};\beta_{\text{eff}}]}^{\text{fs}} (1 - f^{\text{dyn}})$$

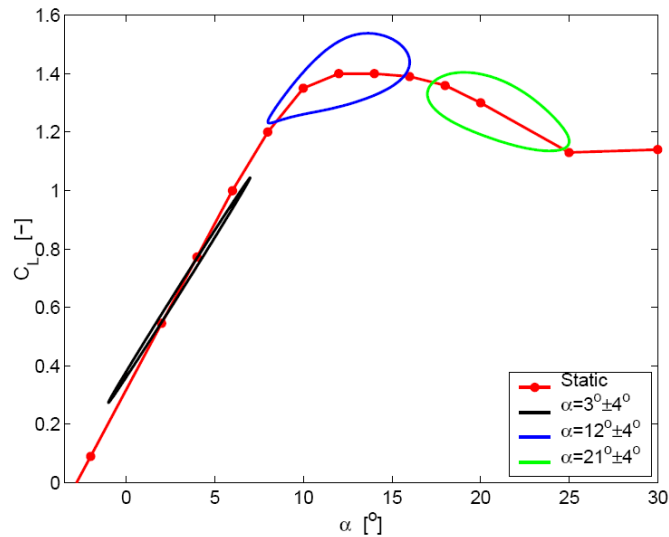


- C_l^{att} and C_l^{fs} computed from steady input data.

- Dynamics from weight, separation function f

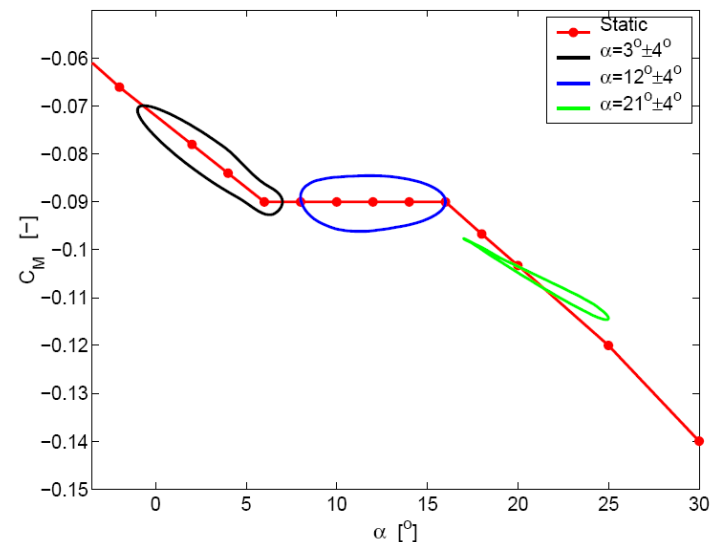
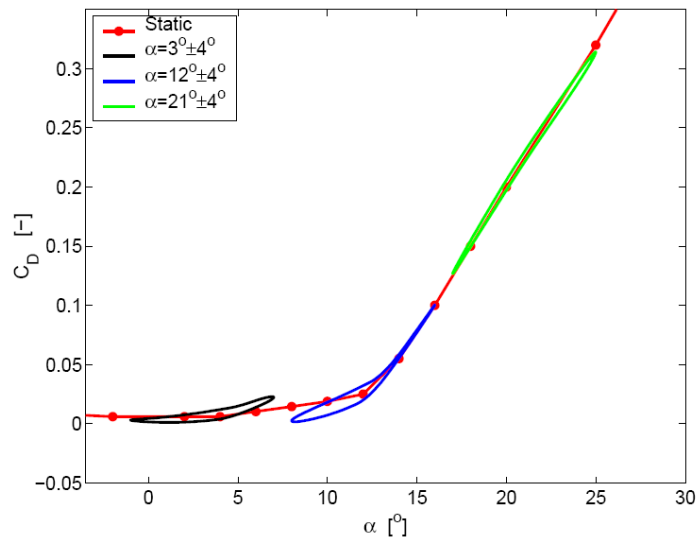


Dynamic effects on both C_l , C_d and C_m

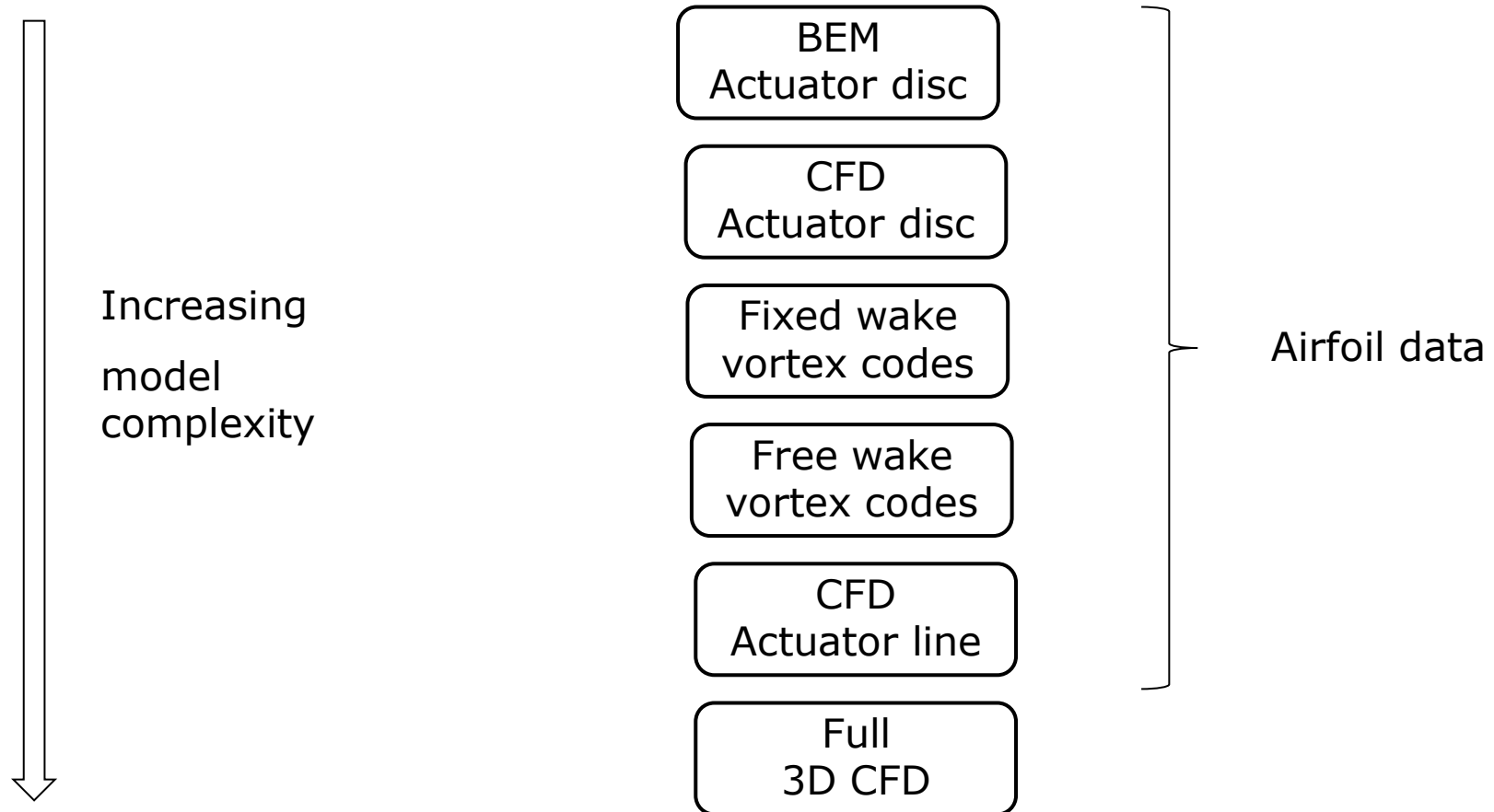


Steady C_l , C_D and C_m data from pc_file

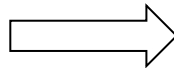
Please use the MHH stall model as default since the Øye model only contains loops on C_l



Induction modelling



Actuator disc

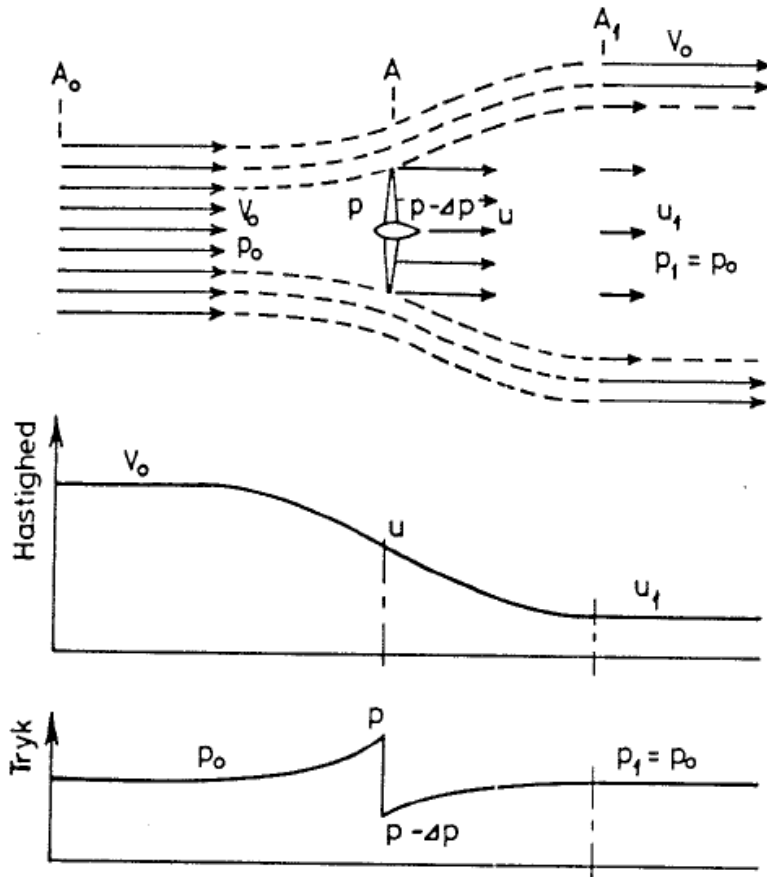


Model of ideal turbine
- Infinitely many blades

**The reaction of the blade forces applied
on the flow as volume or body forces**

**BEM equations (relation between
induction and load) derived for a
constant loaded actuator disc**

The energy conversion – Bernoulli



$$T = \Delta p A$$

Bernoulli

$$\frac{1}{2} \rho V^2 + p = H$$

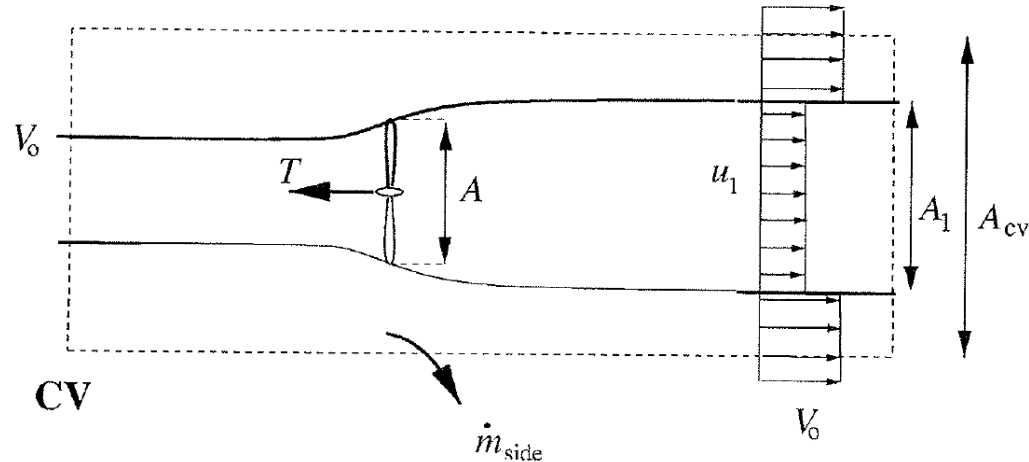
Applied before and after rotor disc

$$p_0 + \frac{1}{2} \rho V_0^2 = p + \frac{1}{2} \rho u^2$$

$$p - \Delta p + \frac{1}{2} \rho u^2 = p_0 + \frac{1}{2} \rho u_1^2$$

$$\Rightarrow \Delta p = \frac{1}{2} \rho (V_0^2 - u_1^2)$$

The energy conversion – 1D momentum



Change in momentum

$$\vec{T} = \dot{m} \Delta \vec{V} \quad \Rightarrow T = \rho V_0 A_{CV} V_0 - \rho u_1 A_1 u_1 - \rho V_0 (A_{CV} - A_1) V_0 - \dot{m}_{side} V_0$$

Conservation of mass and insert in momentum equation

$$\rho A_{CV} V_0 = \rho A_1 u_1 + \rho (A_{CV} - A_1) V_0 + \dot{m}_{side} \quad \Rightarrow \dot{m}_{side} = \rho A_1 (V_0 - u_1)$$

$$\dot{m} = \rho u A = \rho u_1 A_1 \quad \text{yields} \quad T = \dot{m} (V_0 - u_1)$$

Combining momentum and Bernoulli

$$u = \frac{1}{2} (V_0 + u_1)$$

1D momentum continued

Introducing the induction factor a

$$u = (1-a)V_0 \quad u_1 = (1-2a)V_0$$

Extracted power for an ideal rotor is difference in power from inlet to outlet

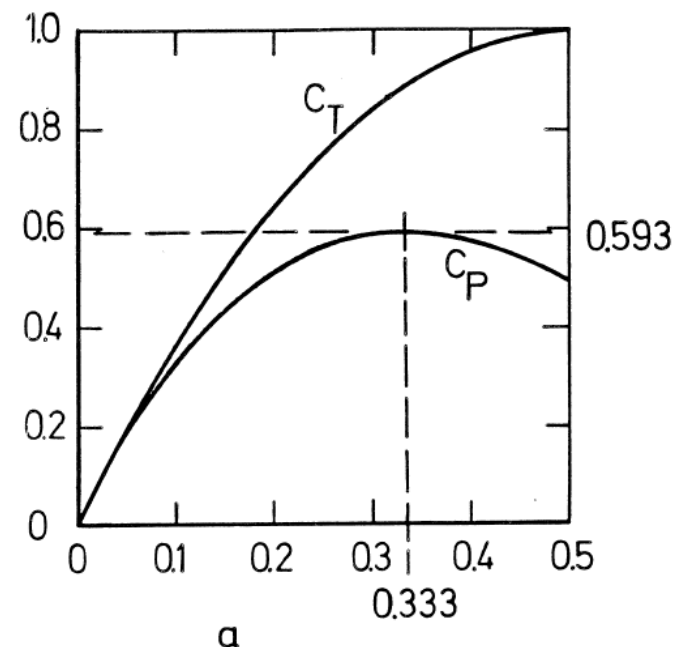
$$P = \frac{1}{2} \dot{m} (V_0^2 - u_1^2)$$

$$C_p = \frac{P}{P_{avail}} = \frac{2\rho A V_0^3 a(1-a)^2}{\frac{1}{2} \rho V_0^3 A} = 4a(1-a)^2$$

Similar for the torque

$$T = \rho u A (V_0 - u_1)$$

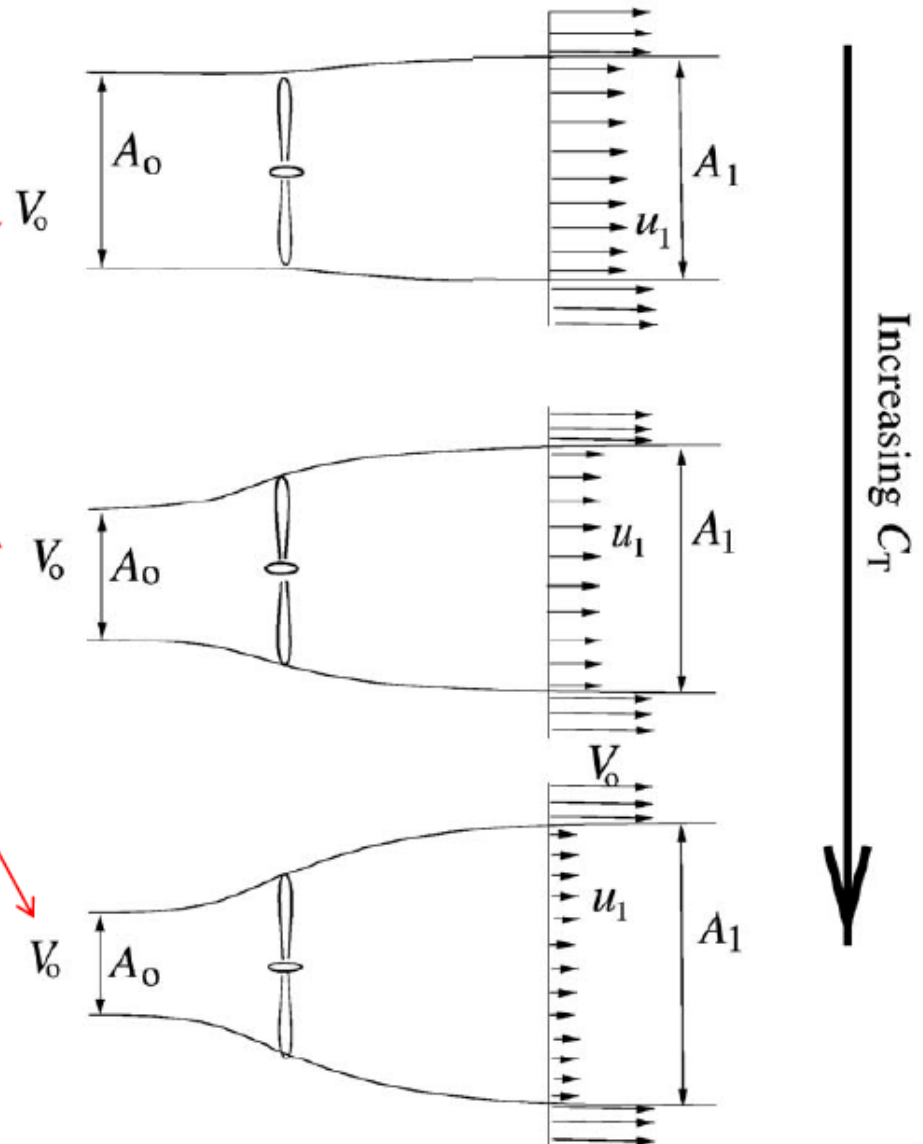
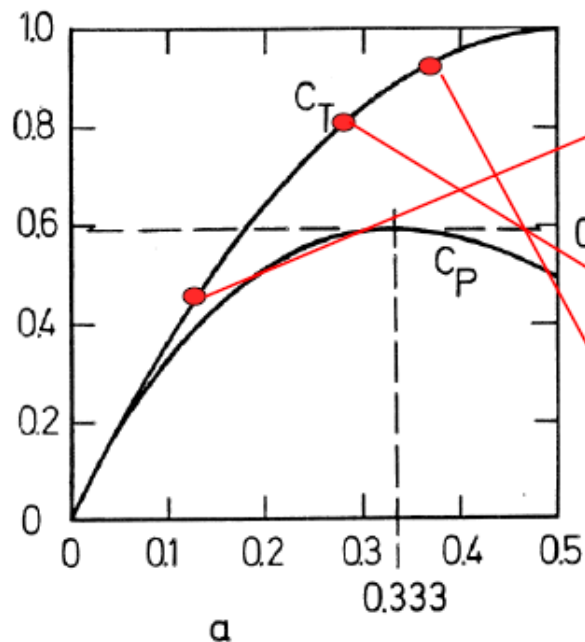
$$C_T = \frac{T}{T_{avail}} = \frac{2\rho A V_0^2 a(1-a)}{\frac{1}{2} \rho V_0^2 A} = 4a(1-a)$$



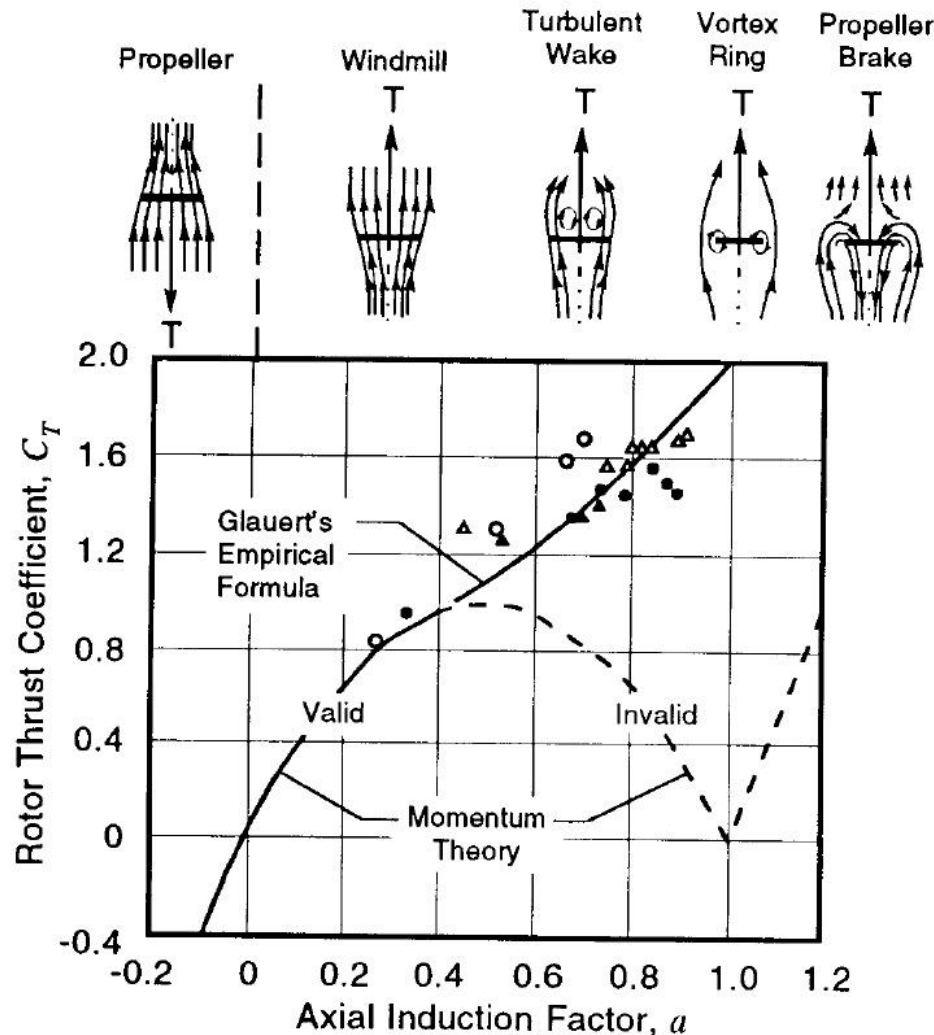
1D momentum continued

Introducing the induction factor a

$$u = (1-a)V_0 \quad u_1 = (1-2a)V_0$$



The relation between thrust and axial induction



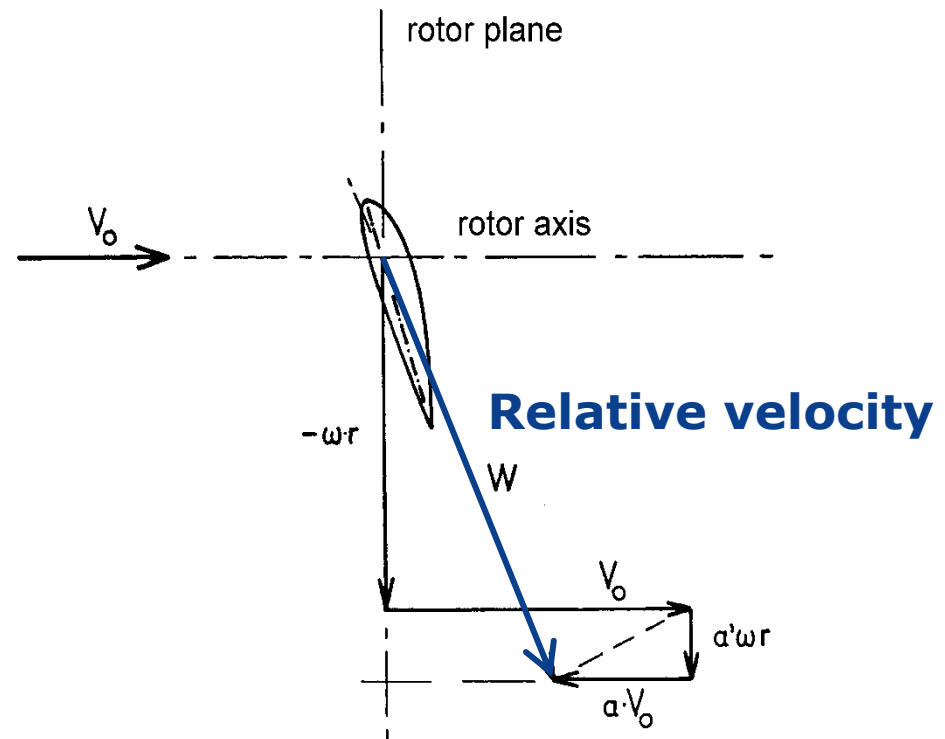
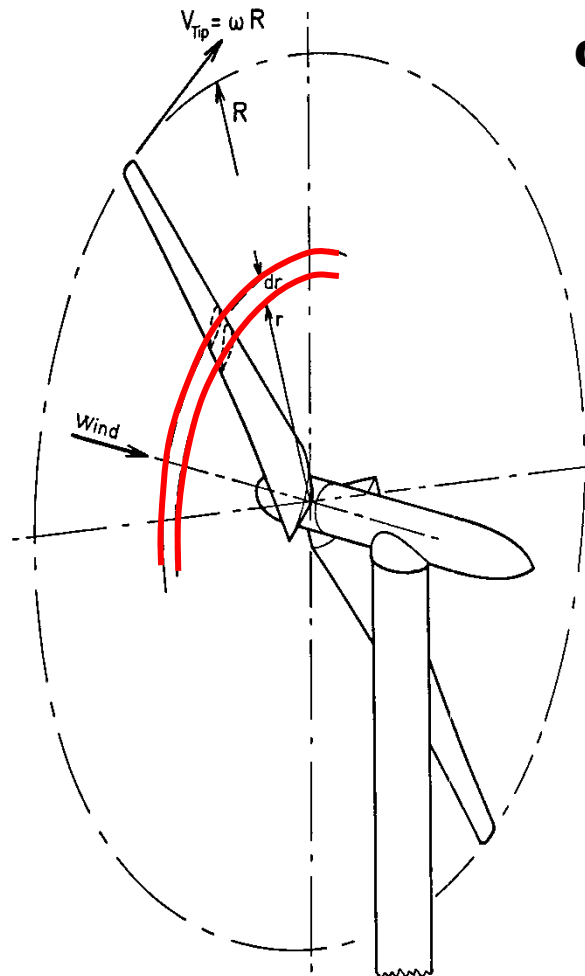
$$u = (1-a)V_0$$

$$C_T = \frac{4a(1-a)}{T}$$

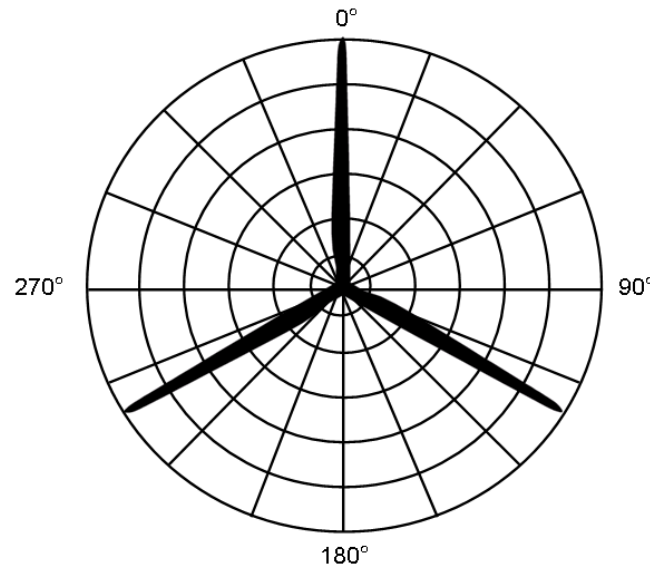
$$C_T = \frac{T}{\frac{1}{2}\rho V_0^2 A}$$

Blade Element Momentum theory

Rotor plane is discretized into
independent concentric annular elements
and azimuthal variation of loading
on which the 1D assumption is applied



Induction model in HAWC2



A non-rotating grid is defined in which induced velocities are calculated. Enables modelling of azimuthal variation of induction e.g. **due to shear** in inflow or **due to turbulence**

BEM in HAWC2 formulated locally in grid.

Blade element theory: $dT = \frac{1}{2} \rho W^2 C_y(\alpha) c N_B$

Axial thrust at given radial station on the blade

$$CT = \frac{dT}{\frac{1}{2} \rho V_\infty^2 2\pi R} \Rightarrow$$

The local thrust coefficient is found

$$CT = \frac{W^2 C_y(\alpha) c N_B}{2\pi r V_\infty^2}$$

V_∞ is the magnitude of the local free wind speed.

$$a \equiv - \frac{u_{induc,axial,local}}{|V_{\infty,local}|}$$

$C_y(\alpha)$ and $C_x(\alpha)$ are projections of $C_l(\alpha)$ and $C_D(\alpha)$
perpendicular and tangential to rotor plane, respectively

BEM in HAWC2 continued

Momentum theory: $CT = 4a(1 - a)$

for $a > 0.33$ the Glauert empirical correlation between a and CT is used

The solution to the above equation is written on the form:

$$a = k_3 CT^3 + k_2 CT^2 + k_1 CT + k_0$$

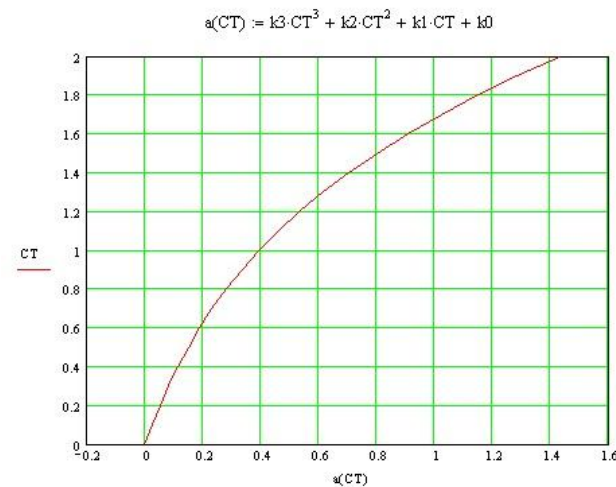
However, the $a = f(CT)$ relation can alternatively be derived from an actuator disc computation on the rotor and would then be a function of position on the rotor disc

$$k(3) = 0.0892074$$

$$k(2) = 0.0544955$$

$$k(1) = 0.251163$$

$$k(0) = -0.0017077$$



Tangential wake rotation

Angular momentum on annular ring

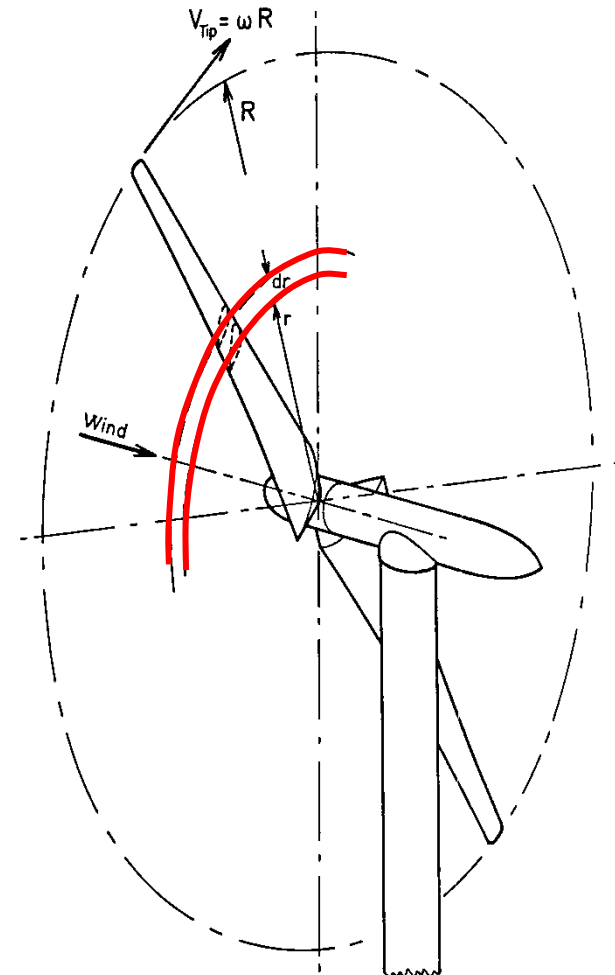
$$dQ = \rho(2\pi r dr)(1-a)V_{\infty}(2a'\Omega)$$

Tangential blade forces on annular ring

$$dQ = \frac{1}{2} \rho W^2 C_x(\alpha) c N_B r dr$$

Above equations solved for the tangential induction factor

$$a' = \frac{W^2 C_x(\alpha) c N_B}{8\pi r^2 (1-a) V_{\infty} \Omega}$$



Sub models to the induction modeling

- ❑ **tip correction model** to account for a finite number of blades
- ❑ a **yaw model** to account for skew inflow
- ❑ a **dynamic induction model** to account for time lag in update of wake due to load changes

Tip loss correction model in HAWC2

The Prandtl tip correction
– take into account finite
number of blades

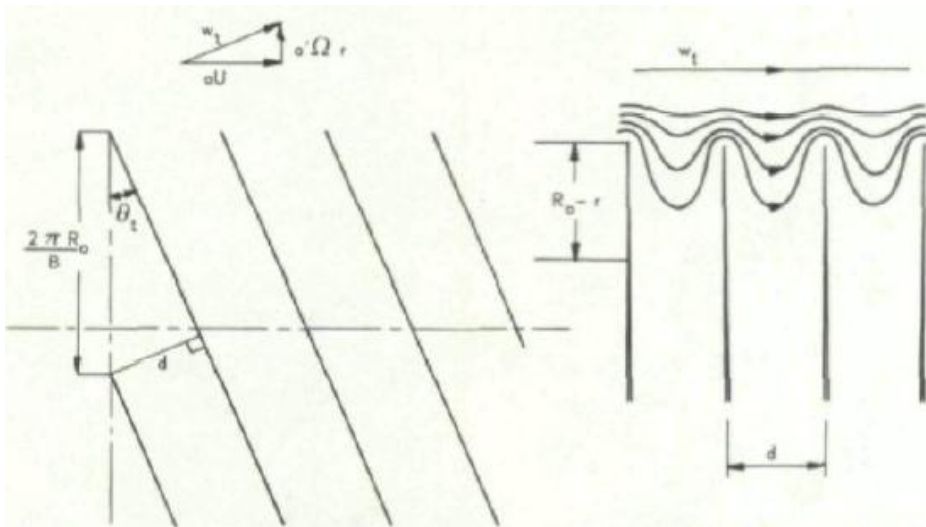
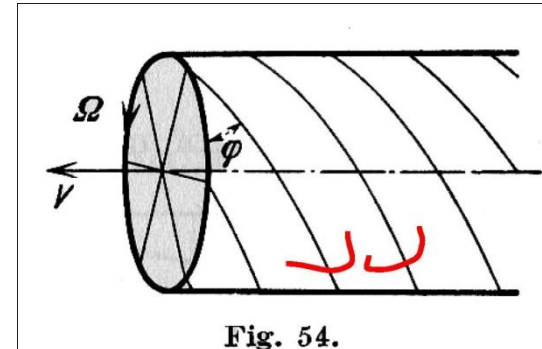


Fig. 4.13 Flow problem solved by Prandtl (Ref. 4.6) to estimate the tip correction factor

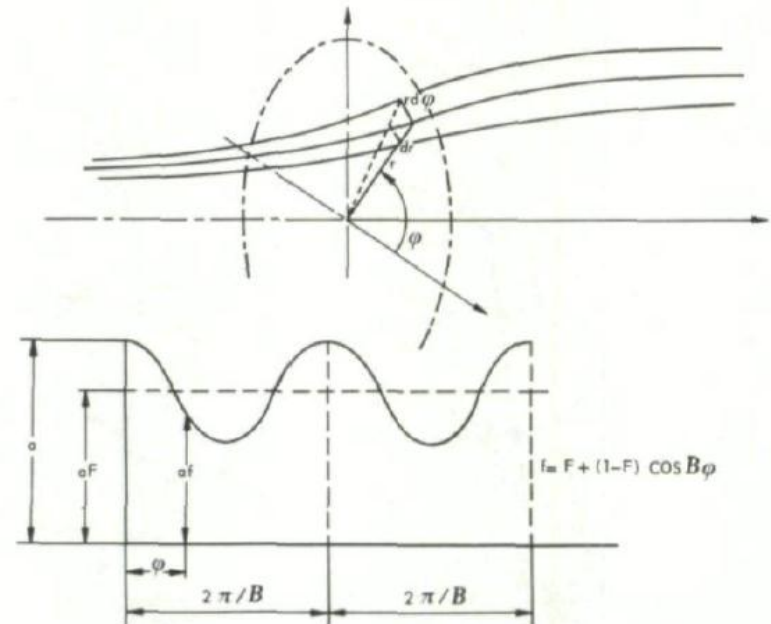


Fig. 4.15 Elemental streamtube through the rotor and the assumed variation of the induced velocity factor with azimuth-angle ϕ

Tip loss correction model in HAWC2

- The tiploss model used in HAWC2 is based on the modified expression by Wilson and Lissaman

$$F = \frac{2}{\pi} \cos^{-1} \left(\exp \left(-\frac{N_B}{2} \frac{(1-r/R)}{(r/R) \sin \phi} \right) \right)$$

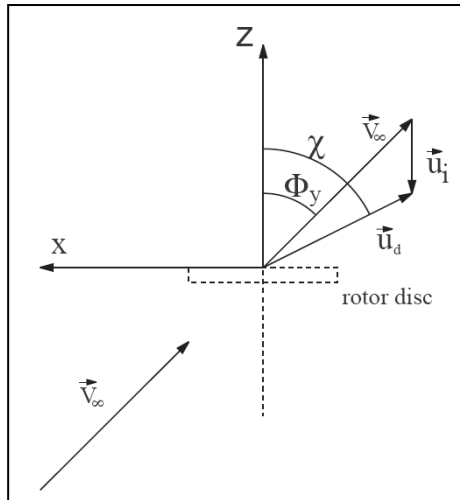
- This factor is inserted in the induction calculation

$$CT = 4a F (1 - a) \quad \text{or} \quad \frac{CT}{F} = 4a(1 - a)$$

$$CT^* = \frac{CT}{F}$$

$$a = k_3 CT^{*3} + k_2 CT^{*2} + k_1 CT^*$$

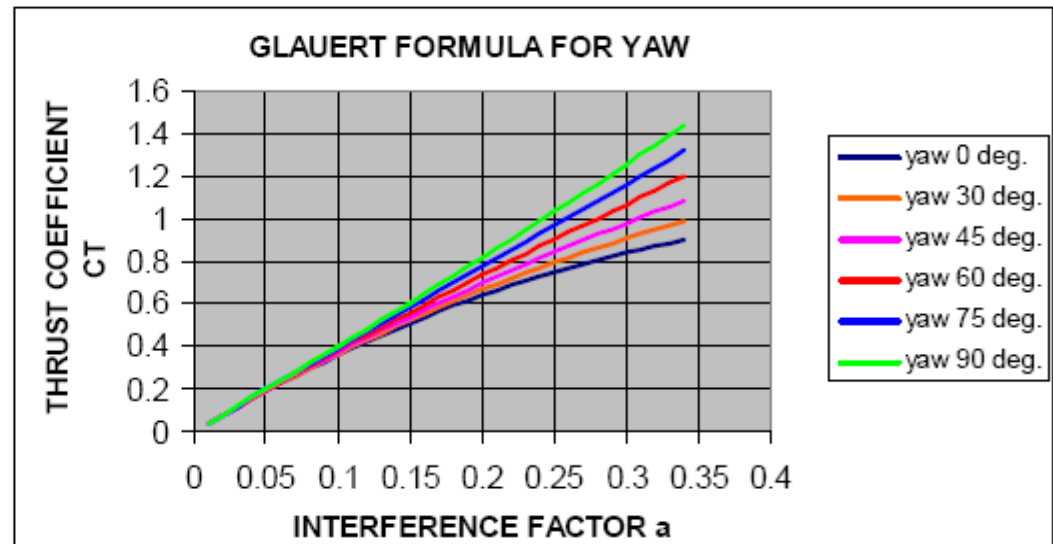
Yaw correction model in HAWC2



Based on model by Glauert

$$T = A\rho|\vec{V}_{\infty} + \vec{u}_i|2u_i$$

$$CT = 4a(1 + a^2 - 2a\cos\Phi_y)^{\frac{1}{2}}$$



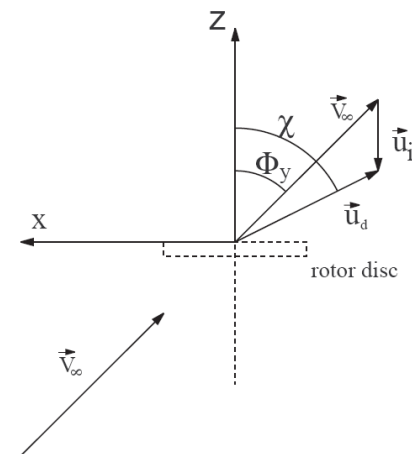
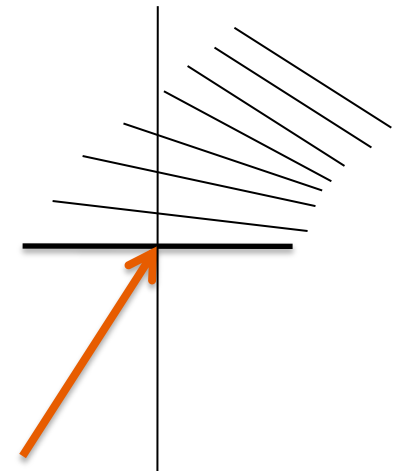
Azimuthal impact on yawed inflow

$$u_{ix} = u_i \left(1 + k_x r \cos(\psi) + k_y r \sin(\psi) \right)$$

ψ Is the azimuth angle

Table 3.1. *Various Estimated Values of First Harmonic Inflow*

Author(s)	k_x	k_y
Coleman et al. (1945)	$\tan(\chi/2)$	0
Drees (1949)	$(4/3)(1 - \cos \chi - 1.8\mu^2)/\sin \chi$	-2μ
Payne (1959)	$(4/3)(\mu/\lambda/(1.2 + \mu/\lambda))$	0
White & Blake (1979)	$\sqrt{2} \sin \chi$	0
Pitt & Peters (1981)	$(15\pi/23) \tan(\chi/2)$	0
Howlett (1981)	$\sin^2 \chi$	0



In HAWC2 the Coleman method is used with a constant of 0.4 instead of 0.5

$$\tan(\chi) = \frac{V_\infty \sin(\Phi_y)}{V_\infty \cos(\Phi_y) - u_i}$$

Yawed inflow

The reduction K_a of the induction factor a as function of yaw angle and thrust coefficient was shown in the figure above and in order to use the correction directly in the induction calculation the following polynomial fit has been derived:

$$K_a = CT^3 k_3 + CT^2 k_2 + CT k_1$$

where the parameters k_3 , k_2 and k_1 depends on the yaw angle (radians) in the following way:

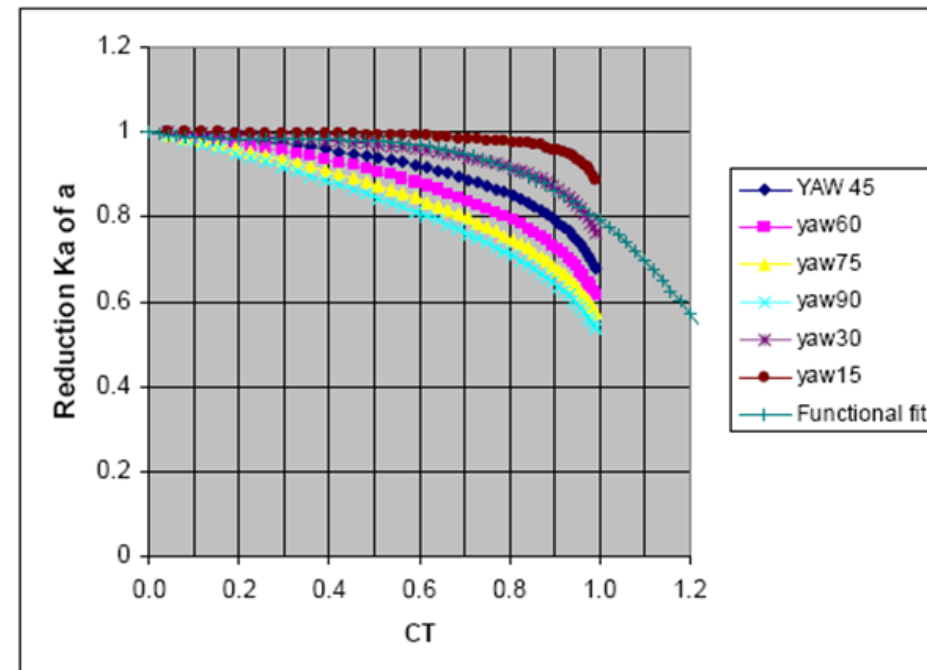
$$k_3 = -0.6481\Phi_y^3 + 2.1667\Phi_y^2 - 2.0705\Phi_y$$

$$k_2 = 0.8646\Phi_y^3 - 2.6145\Phi_y^2 + 2.1735\Phi_y$$

$$k_1 = -0.164\Phi_y^3 + 0.4438\Phi_y^2 - 0.5136\Phi_y$$

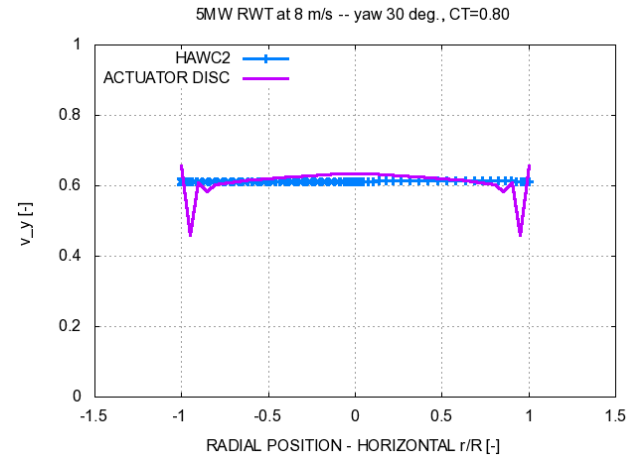
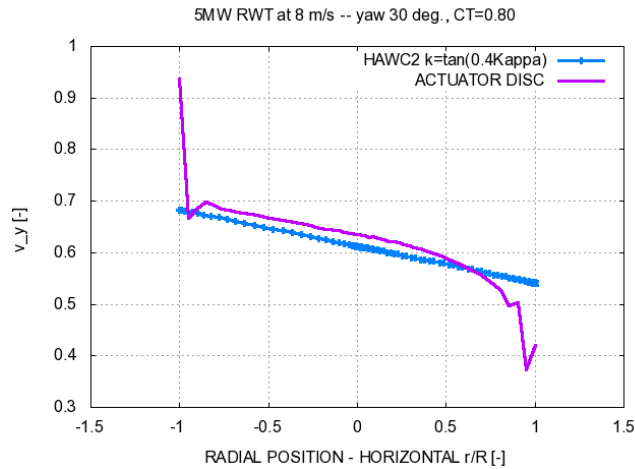
Modification of induction

$$a_{cor} = K_a a$$

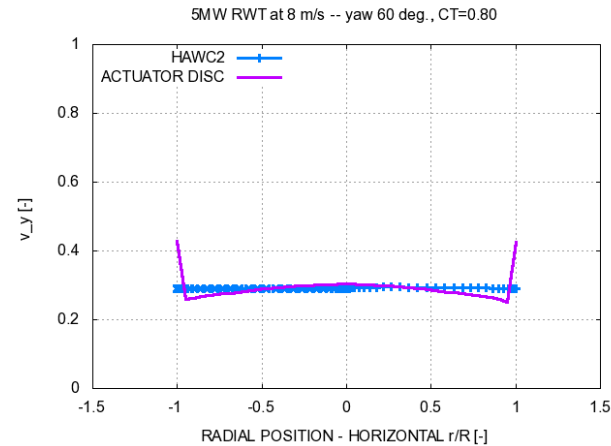
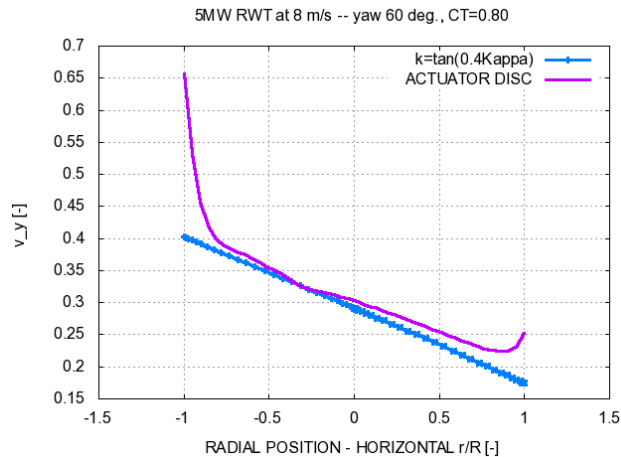


Comparison between HAWC2 and CFD actuator disc computations

30deg



60deg



Dynamic inflow model in HAWC2



Diameter: 100m

Rotor area: 7854m²

Density: 1.225 Kg/m³

Wind speed: 14m/s

Massflow: 135t/s (!)

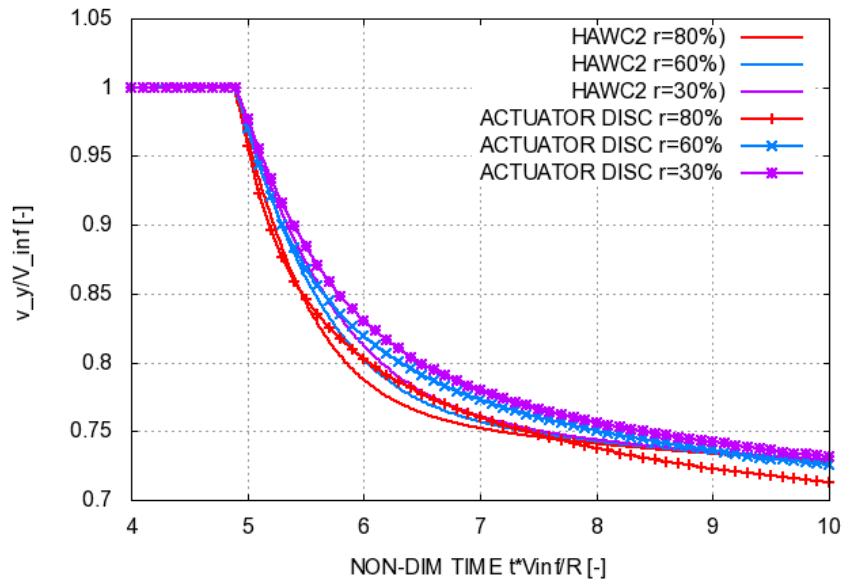


Lockheed planes: takeoff weight 130-140 t

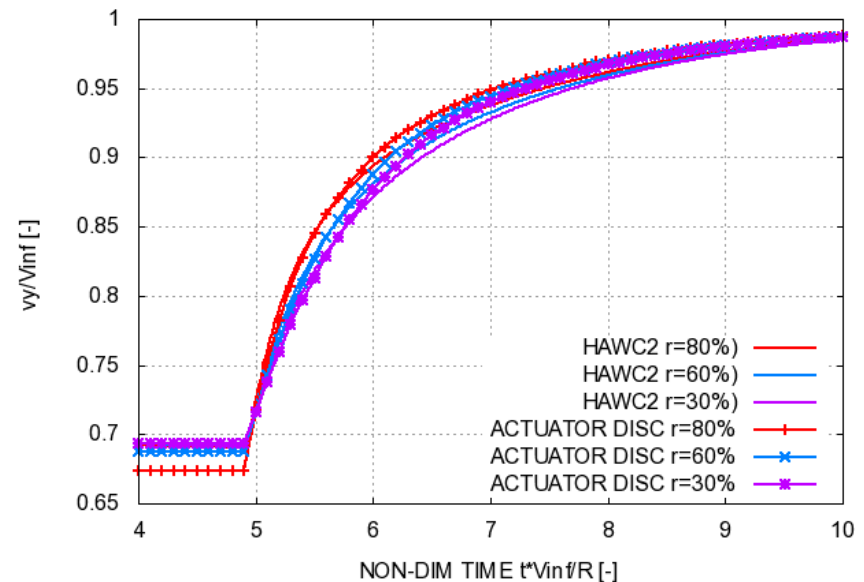
It takes considerable time to obtain a flow equilibrium!

Effect of dynamic inflow – numerical actuator disc simulations – step change in loading

5MW RWT at 10 m/s -- Step in CT from 0 to 0.89



5MW RWT at 10 m/s -- Step in CT from 0.89 to 0



Effect of dynamic inflow – numerical actuator disc simulations – step change in loading

$$\tilde{\tau} = \tau \frac{V_{\infty}(1 - 1.5a)}{R}$$

In both the BEM and the NW model, this is the approach used, and as will be seen later in the paper, this assures that the time constants reflects the slower development of the wake when high induction are present. The effect of changing the normalization can be seen in Figure 1, where the use of the actual wake velocity makes the curves collapse.

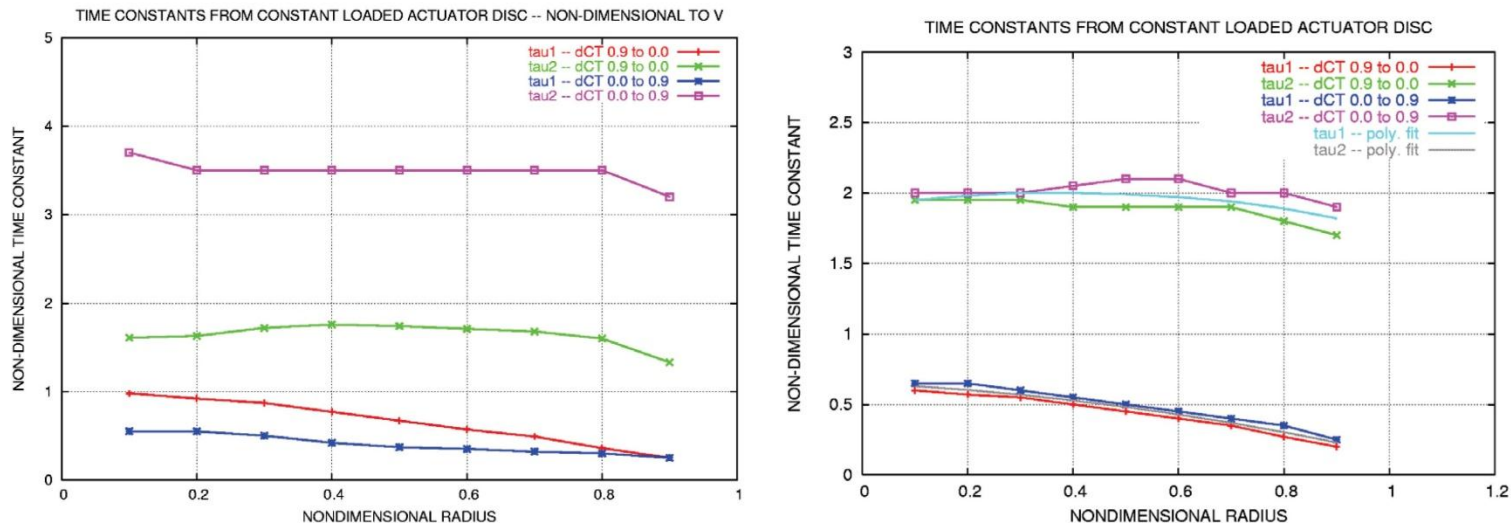


Figure 1: The dependency of the non-dimensionale time constant on the choice of normalization velocity used, left figure using the free stream velocity, right figure shows the use of the actual wake velocity.

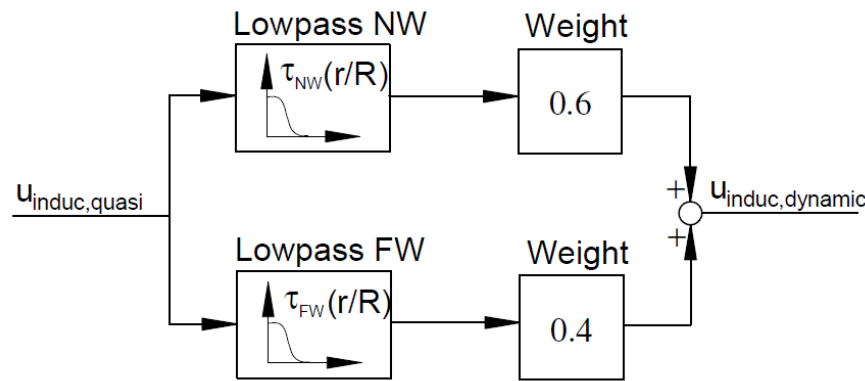
Dynamic inflow

$$\tau_{NW}^* \left(\frac{r}{R} \right) = k_2 \left(\frac{r}{R} \right)^2 + k_1 \left(\frac{r}{R} \right)^1 + k_0$$

	<i>NW</i>	<i>FW</i>
k2	-0.4783	-0.4751
k1	0.1025	0.4101
k0	0.6125	1.9210

Dynamic inflow model in HAWC2

The Dynamic inflow is handled by two first order filters coupled in parallel with weight factors



$$\tau_{FW}\left(\frac{r}{R}\right) = \tau_{FW}^*\left(\frac{r}{R}\right) \frac{R}{V_{\infty,y} \text{ MAX} \left[1 + 3 \frac{V_{induc,y}}{V_{\infty,y}}, 0.2 \right]}$$

$$\tau_{NW}\left(\frac{r}{R}\right) = \tau_{NW}^*\left(\frac{r}{R}\right) \frac{1.8R}{V_{\infty,y} \text{ MIN} \left[1 - 3 \frac{V_{induc,y}}{V_{\infty,y}}, 2.0 \right]}$$

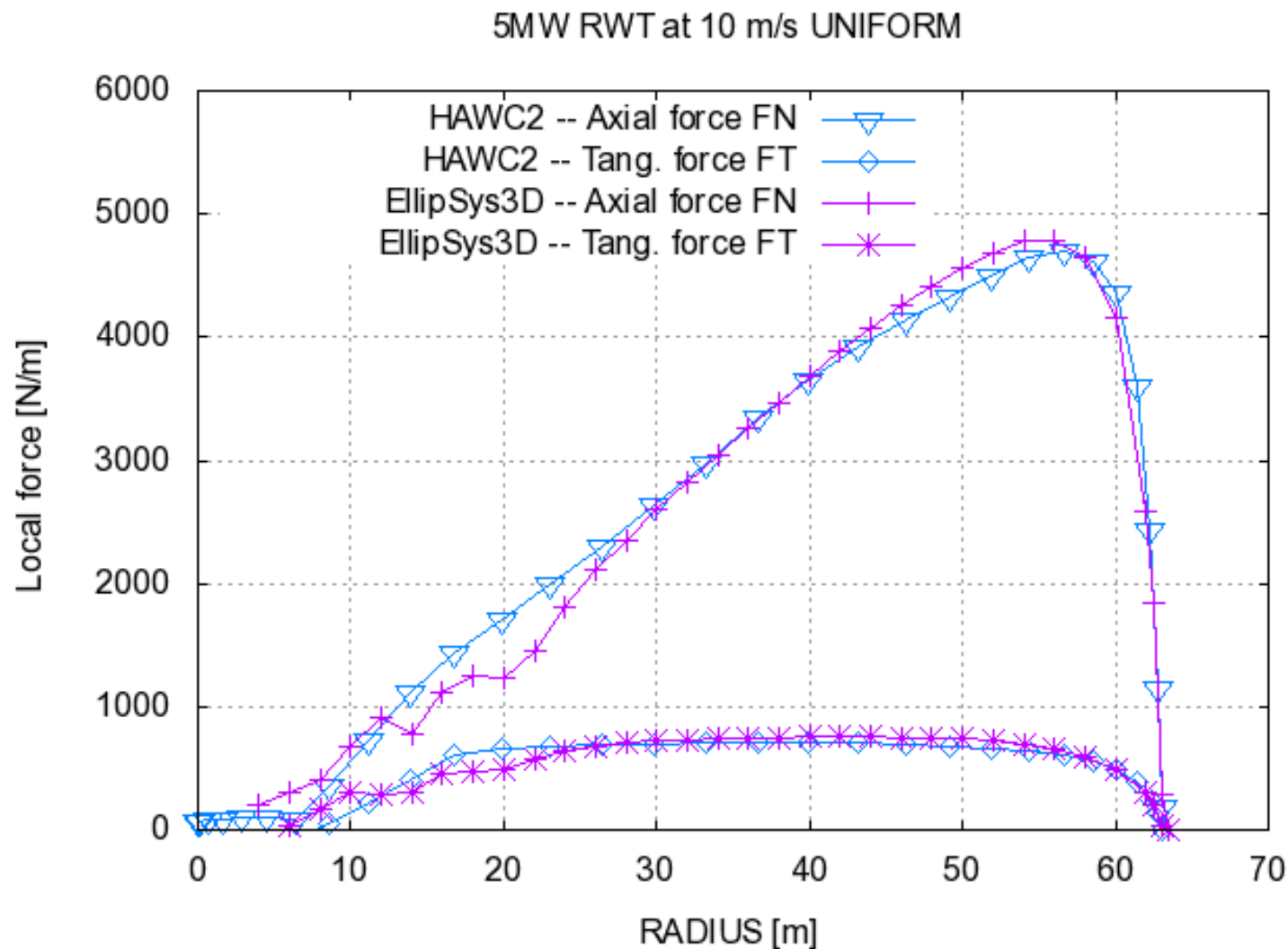
The non-dim time constants was found to approximately 2.0 for the far wake contribution and 0.5 for the near wake part

BEM procedure

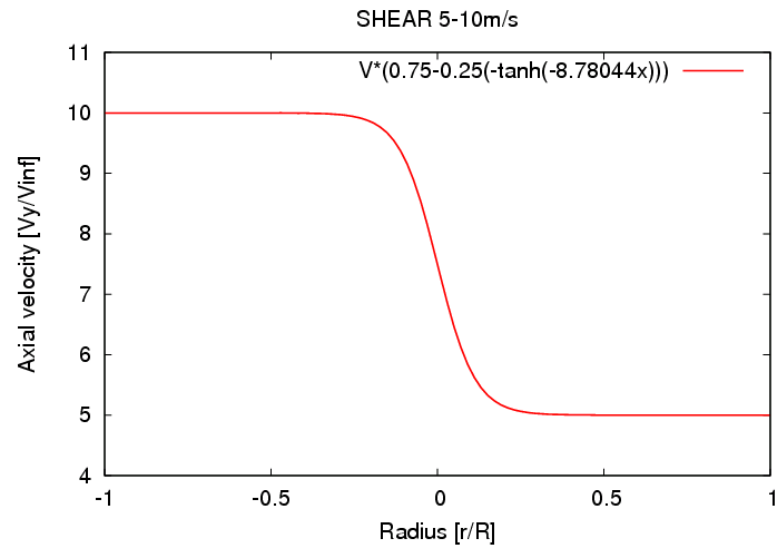
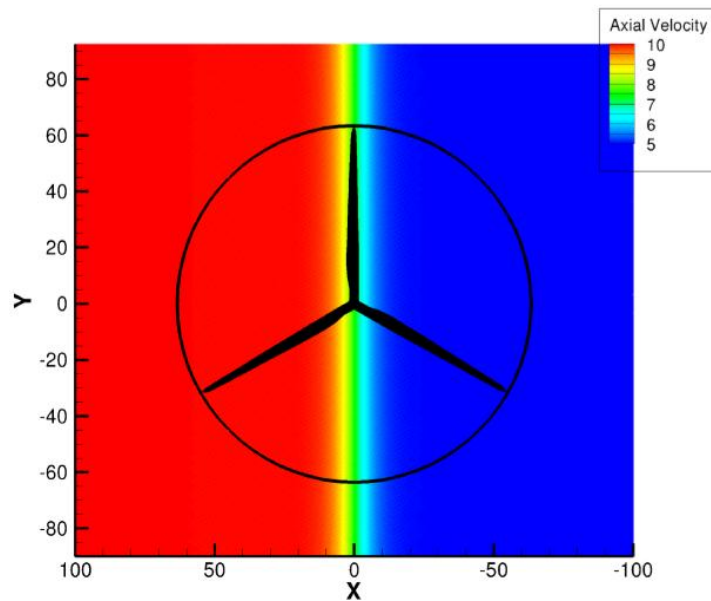
- For each point in the polar grid
 1. Get the local wsp and use local induced wsp from previous timestep/iteration
 2. Get the local CT calculated at the two neighboring blades if the local wsp and induction was used there.
 3. Account for Prandtl tiploss
 4. Interpolate CT based on azimuthal distance between grid point and the two blades. Now the local CT is known.
 5. Calculate local induction factor a
 6. Correct mean level of a based on skew inflow angle.
 7. Calculate tangential induction factor a_m
 8. Calculate induced wind speeds axial (1) and tangential
 9. Correct for azimuthal variations of axial induction u related to skew inflow angle incl. axial induction influence..
 10. Update u and u_t in time based on two first order low pass filters. One for near wake contribution and one for far wake.
- For each point on the blade
 11. Get the induced velocity based on azimuthal interpolation of the two closest grid points at same radius.

Validation results

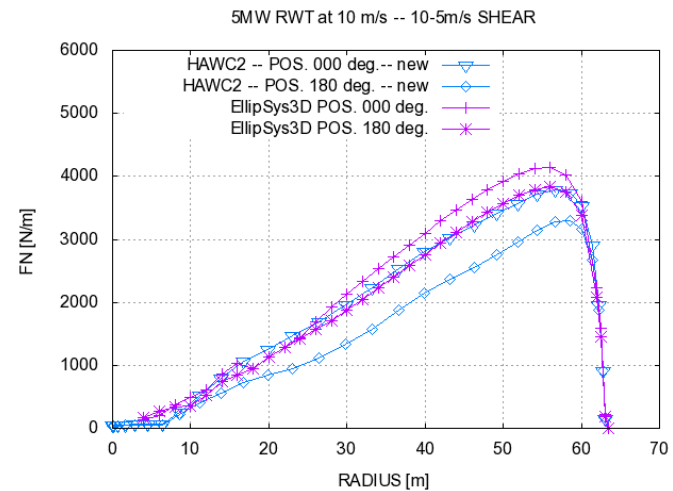
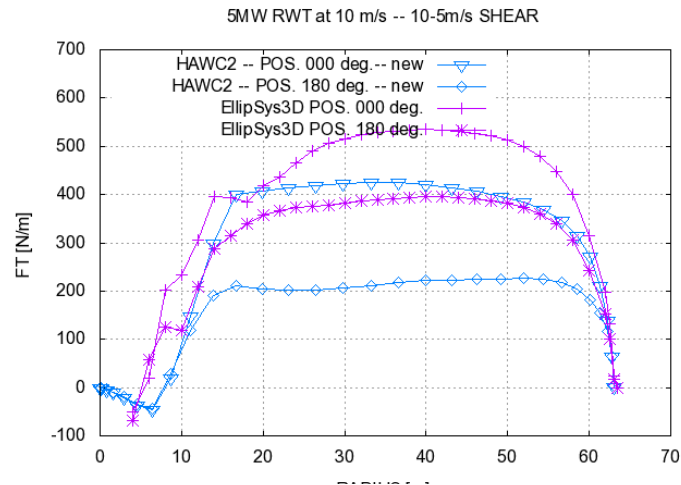
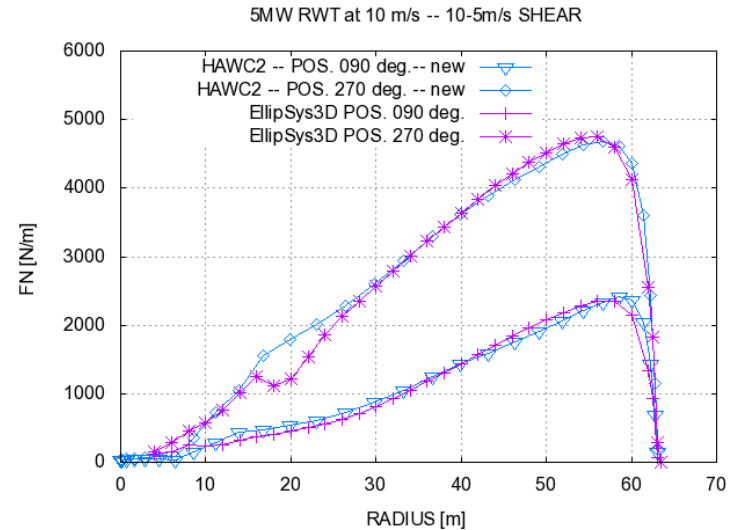
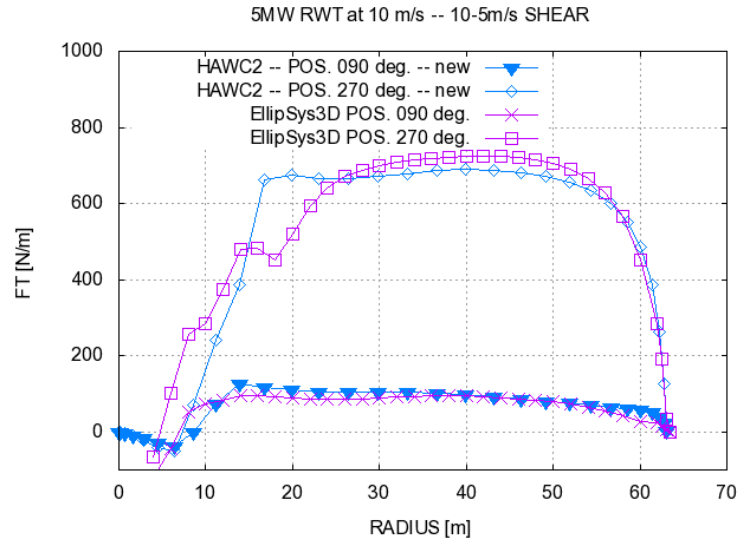
Comparison



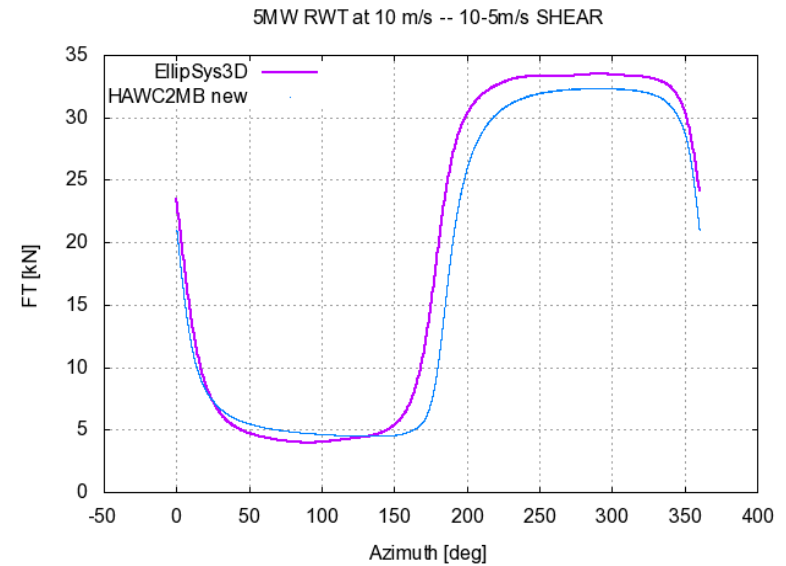
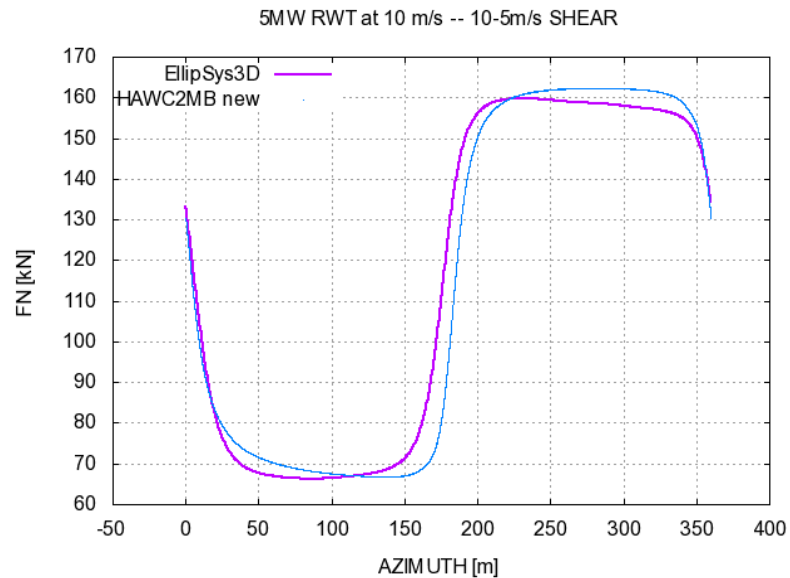
Special shear (1)



Special shear (2)

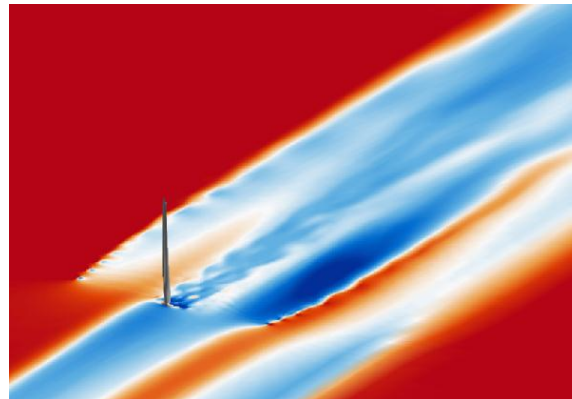
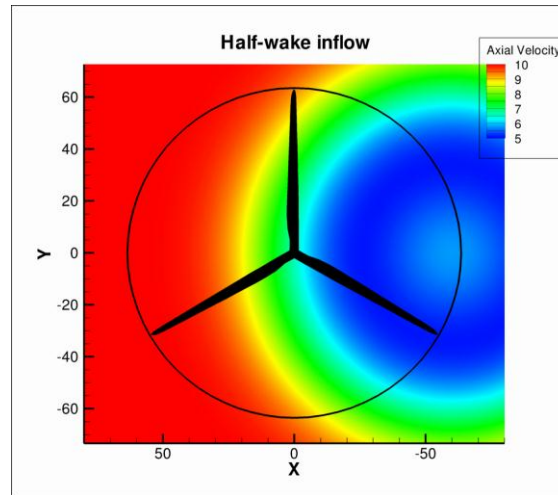
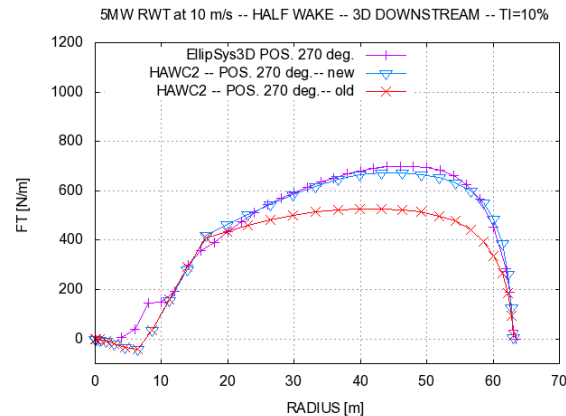
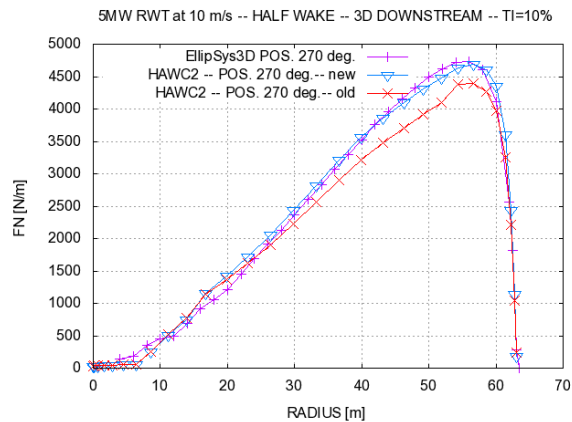


Special shear (3)

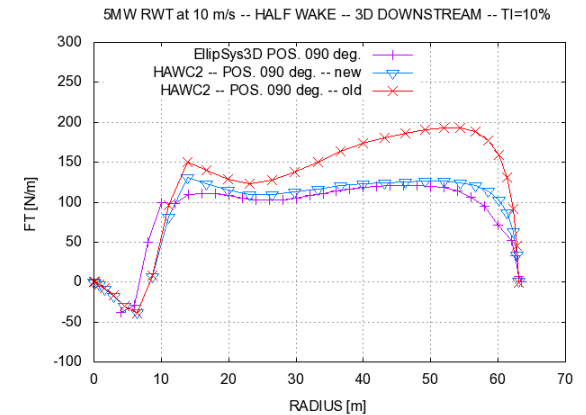
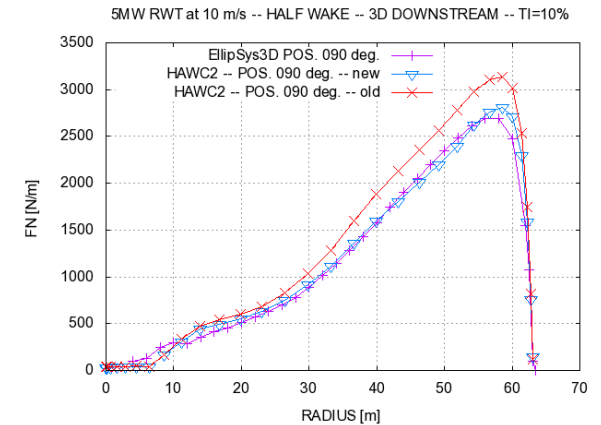


Wake situation

270°

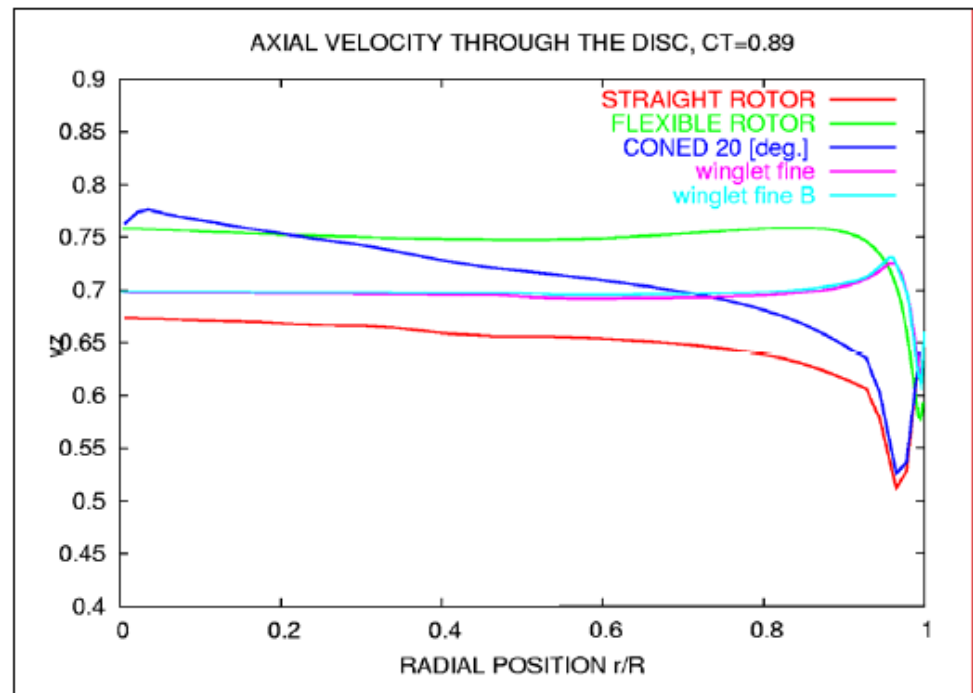
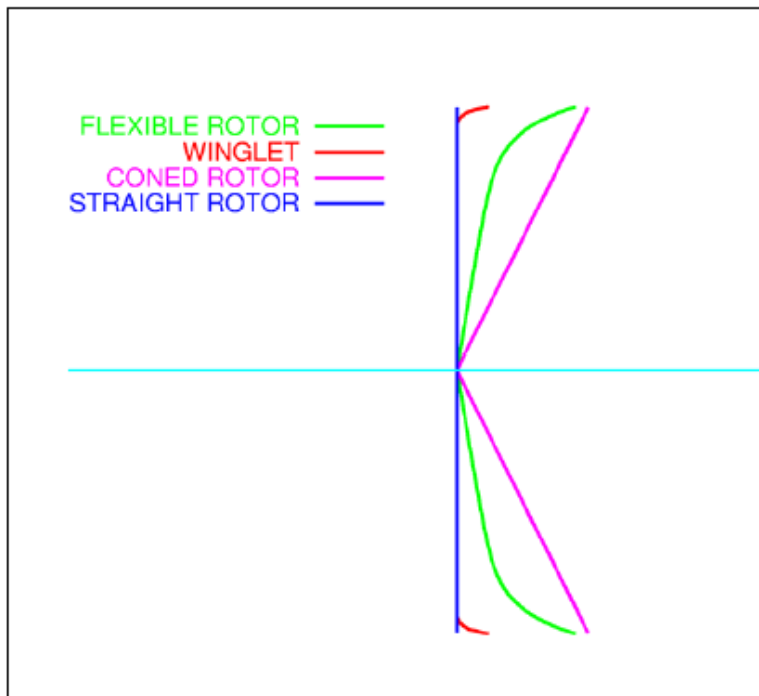


90°



Examples of limitations of the BEM model

All rotors with the same uniform loading, $CT=0.89$



For such special rotors a user defined a - CT relation can be used if the characteristic is known from e.g. CFD actuator disc simulations

Aerodynamic input in HAWC2

```
begin aero ;
  nblades 3;
  hub_vec hub_center 3 ; rotor rotation vector (normally shaft composant
                        ; directed from pressure to sustion side)
  link 1 mbdy_c2_def blade1;
  link 2 mbdy_c2_def blade2;
  link 3 mbdy_c2_def blade3;
  ae_filename      ./data/H2_blade_ae.dat ;
  pc_filename      ./data/H2_blade_pc.dat ;
  induction_method 1 ;      0=none, 1=normal
  aerocalc_method  1 ;      0=no aerodynamic, 1=with aerodynamic
  arosections      30 ;
  ae_sets          1 1 1;
  tiploss_method   1 ;      0=none, 1=prandtl
  dynstall_method  2 ;      0=none, 1=stig øye method,2=hhh method
end aero ;
```

2D aerodynamic inputs: the ae file

Aerodynamic properties depend on geometric characteristics:

- Chord length
- Profile geometry -> steady aerodynamic forces (complex relation, input: *profile coefficient tables*)

Input: *aerodynamic layout* of the blade.

Given in the `ae_filename` file:

```
1 Aerodynamic planform
1 12
0      3.5      100      1
0.5    3.5      100      1
2.1    3.5      99.9734  1
4.9    4.2      74.46957 1
7.7    4.9      48.96571 1
10.5   5.6      35.71321 1
21.7   4.9014   20.58616 1
32.9   3.5489   17.32931 1
35.7   3.24968  16.76627 1
47.033 1.83467  15.65677 1
52.26  0.913    14.99     1
52.9   0.8      14.7875  1
```

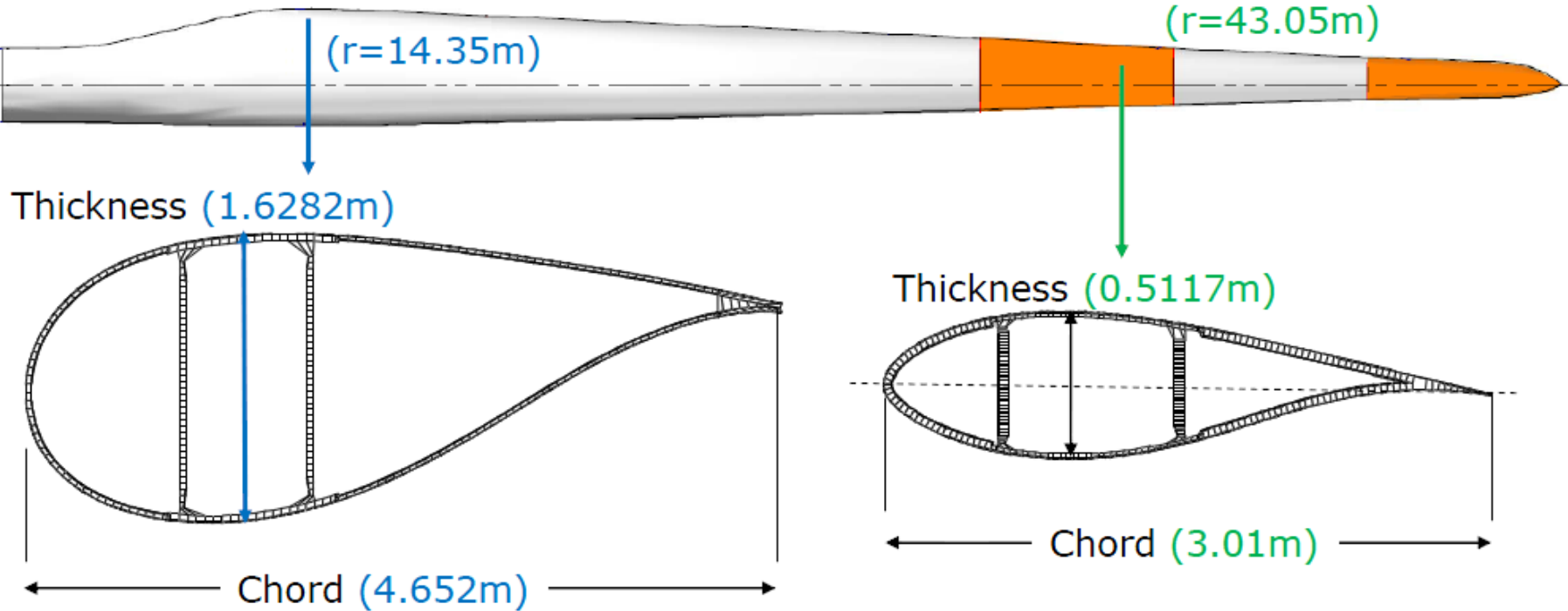


1. Total number of datasets given in the file
2. Set Id number , Tot. number of points given in the set

In the following lines:

- Spanwise position [m]
- Chord Length [m]
- Thickness/Chord ratio [%] (link to prof. coeff. tables)
- Profile Coefficient Set

Step 1: \data\NREL_5MW_ae.txt



\data\NREL_5MW_ae.txt

```
1 Chord[m] T/C[%] Set no.
1 19 Blade
...
14.350 4.652 35.000 1
43.050 3.010 17.000 1
...
```

Thickness/Chord

Profile Coefficients: input file

Profile coefficients collected in the `pc_filename` file:

```
1 Airfoil data for the nrel 5 mw turbine
```

```
8
```

```
1 127 17.0 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
```

```
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
```

```
2 127 21.0 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
```

```
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
```

```
8 127 100 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
```

```
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
```

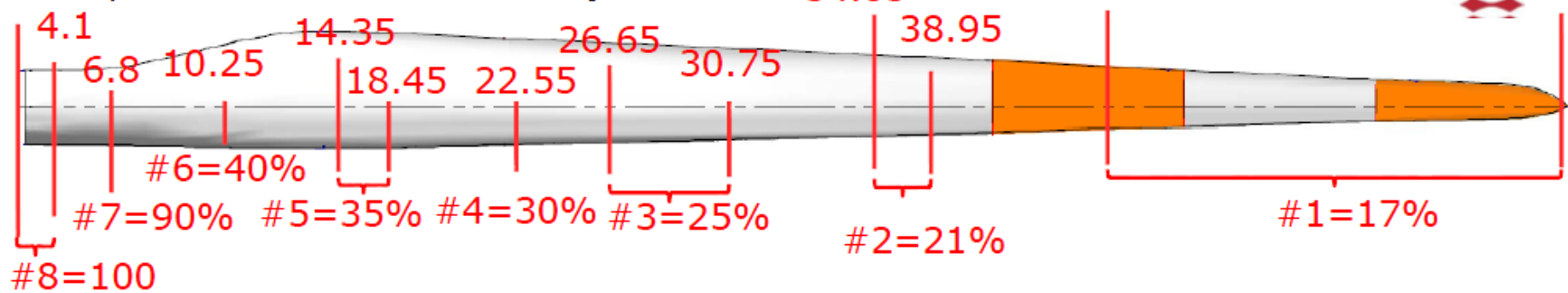
1. Total number of "sets"
2. Number of profiles in the first set.
3. First profile in the first set:
Index | N rows | t/c profile (*link to ae*)

NB: The t/c in *increasing* order

Then for each profile, steady coefficients table:

- Aoa [deg] (arbitrary spacing)
- Cl
- Cd
- Cm

Step 2: \data\NREL_5MW_pc.txt



\data\NREL_5MW_pc.txt

```
1 Airfoil data for the nrel 5 mw turbine
8
1 127 17.0 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
2 127 21.0 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
8 127 100 comments here...
-180.00 0.000 0.0198 0.0000
-175.00 0.374 0.0341 0.1880
+175.00 0.749 0.0955 0.3770
+180.00 0.659 0.2807 0.2747
```

1. Total number of "sets"
2. Number of profiles in the first set.
3. First profile in the first set:

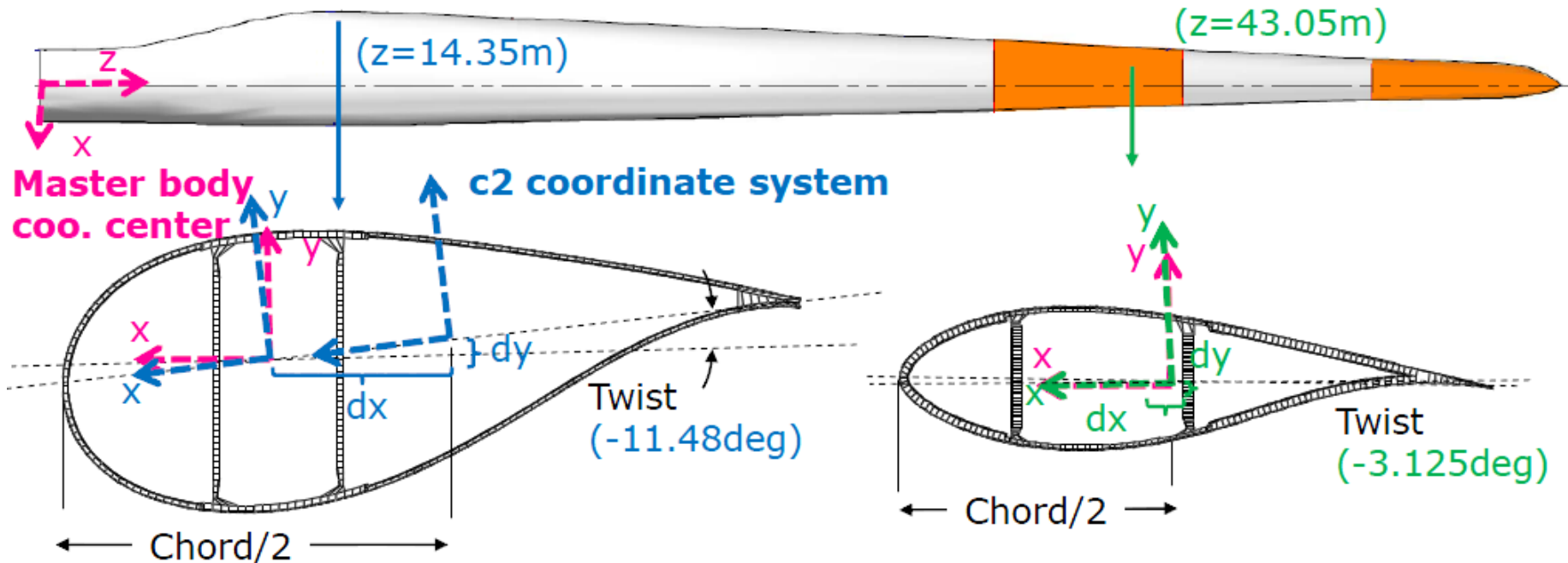
Index | N rows | t/c profile (*link to ae*)

NB: The t/c in *increasing* order

Then for each profile, steady coefficients table:

- Aoa [deg] (arbitrary spacing)
- Cl
- Cd
- Cm

Step 3: \htc\structure_aero.htc



\htc\structure_aero.htc

```

begin main_body;
  name blad1;
...
  begin c2_def;
    nsec 19 ;
...
    ; "dx"    "dy"    "z"    "twist"
    sec 6  -0.5699  0.1157  14.350  -11.480 ;
    sec 13 -0.3757  0.0205  43.050  -3.125 ;
...
  end c2_def ;
end main_body;

```

Can also be used to add

- prebend (dy, dy)
- sweep (dx, dx)

Thank you for
your attention

Corrections for yawed inflow – extra info

General case of momentum for rotor, Glauert

$$\vec{T} = \dot{m} \Delta \vec{V} = A \rho |\vec{V}_\infty + \vec{u}_i| \left(\vec{V}_\infty - (\vec{V}_\infty + 2\vec{u}_i) \right) \Rightarrow \vec{T} = A \rho |\vec{V}_\infty + \vec{u}_i| (-2\vec{u}_i)$$

Which for axial flow is

$$T_z = A \rho |\vec{V}_\infty + \vec{u}_i| (-2u_{i,z}) \Rightarrow T_z = A \rho |u_d| (-2u_{i,z})$$

Geometry relation

$$|u_d|^2 = \left(|V_\infty| \cos \Phi + u_{i,z} \right)^2 + \left(|V_\infty| \sin \Phi \right)^2$$

$$\Rightarrow |u_d|^2 = |V_\infty|^2 \left(1 + \frac{u_{i,z}^2}{|V_\infty|^2} + \frac{2 \cos \Phi u_{i,z}}{|V_\infty|} \right)$$

$$a \equiv - \frac{u_{i,z}}{|V_{\infty, local}|}$$

$$T_z = \rho A \sqrt{|V_\infty|^2 \left(1 + \frac{u_{i,z}^2}{|V_\infty|^2} + \frac{2 \cos \Phi u_{i,z}}{|V_\infty|} \right)} (-2u_{i,z})$$

$$T = \frac{1}{2} A \rho V_0^2 4a(1 + a^2 - 2a \cos \Phi)^{\frac{1}{2}}$$

$$CT = \frac{T}{\frac{1}{2} A \rho V_0^2} = 4a(1 + a^2 - 2a \cos \Phi_y)^{\frac{1}{2}}$$

