

## HAWC2

Hydrodynamic modeling  
Torben J. Larsen

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x) \quad \Delta \int_a^b \epsilon \Theta^{\sqrt{17}} + \Omega \int \delta e^{i\pi} = \{2.7182818284\}$$

Risø DTU  
National Laboratory for Sustainable Energy

## Hydrodynamic model in HAWC2

### Hydrodynamic kinematics:

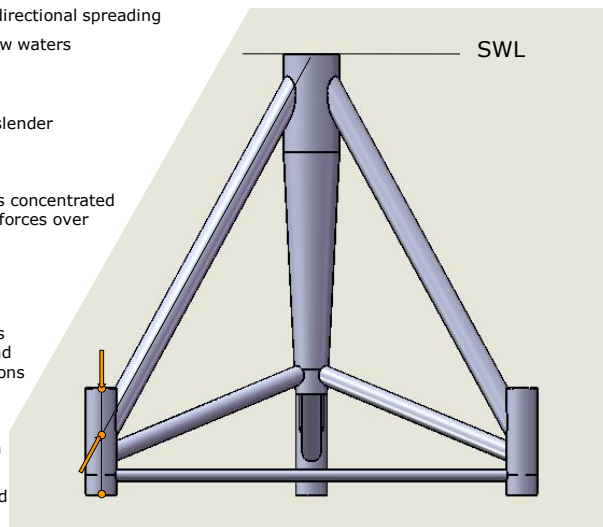
- Regular and irregular Airy waves, directional spreading
- Wheeler stretching used for shallow waters

### Hydrodynamic loads:

- Morison's formula (assumption of slender elements)
- Axial damping term in end nodes
- Axial dynamic pressure inserted as concentrated force on end nodes and distributed forces over conical sections

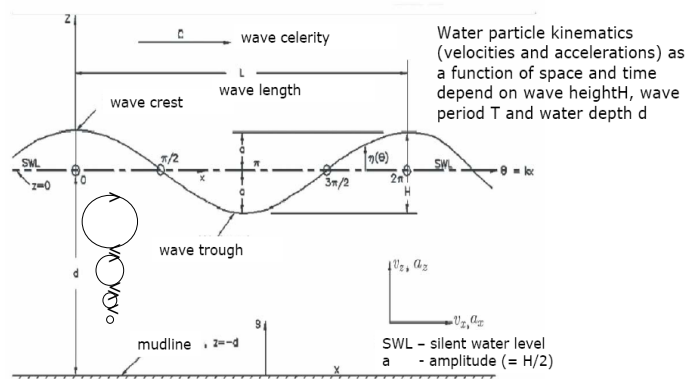
### Buoyancy loads from static pressure:

- Axial dynamic pressure inserted as concentrated force on end nodes and distributed forces over conical sections
- Distributed perpendicular force contribution on angled elements
- Restoring moments distribution on conical sections
- Influence of flooded water included



# Wave kinematics

## Linear wave



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## Linear wave

The elevation is described as a sinus function

$$\eta_i(t, x) = A_i \sin(\omega_i t - k_i x + \phi_i)$$

Where the relation between wave number and frequency is given by the dispersion relation

$$\omega_i^2 = g k_i \tanh(k_i z_0)$$

The horizontal and vertical velocities

$$u_i(t, x, z) = \omega_i \frac{\cosh[k_i(z + z_0)]}{\sinh[k_i z_0]} \eta(t, x) \quad w_i(t, x, z) = \omega_i \frac{\sinh[k_i(z + z_0)]}{\sinh[k_i z_0]} A_i \cos(\omega_i t - k_i x + \phi_i)$$

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## Linear wave (2)

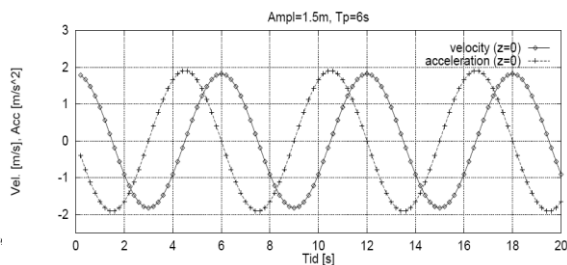
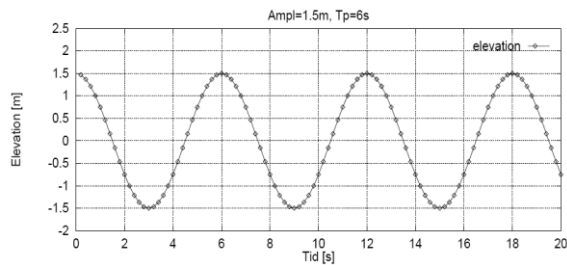
The particle accelerations

$$\dot{u}_i(t, x, z) = \omega_i^2 \frac{\cosh[k_i(z + z_0)]}{\sinh[k_i z_0]} A_i \cos(\omega_i t - k_i x + \varphi_i) \quad \dot{w}_i(t, x, z) = -\omega_i^2 \frac{\sinh[k_i(z + z_0)]}{\sinh[k_i z_0]} \eta(t, x)$$

And the dynamic variation in pressure

$$p_i(t, x, z) = \rho g \frac{\cosh[k_i(z + z_0)]}{\cosh[k_i z_0]} \eta(t, x)$$

## Linear wave (3)



## Wheeler stretching

The velocity profile is extrapolated to the actual surface level by a stretching technique known as Wheeler stretching.

$$z' = \frac{z + d}{1 + \frac{\eta(t)}{d}}$$

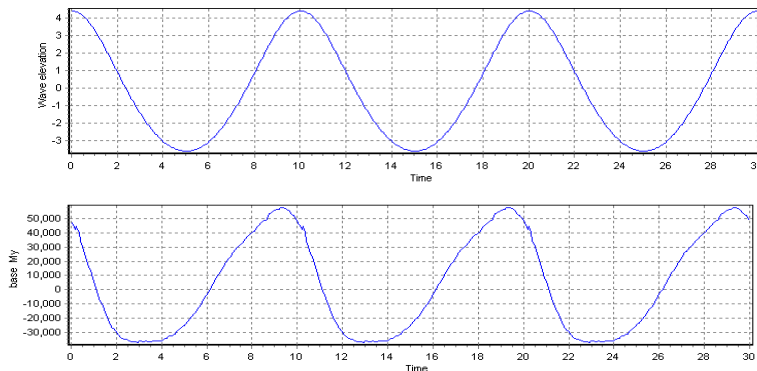
This is inserted in the calculation of velocities instead of  $z+d$ . It ensures that the velocities calculated at  $z=0$  without stretching is inserted on the structure at the elevation level.

$$u_i(t, x, z) = \omega_i \frac{\cosh[k_i(z')]}{\sinh[k_i z_0]} \eta(t, x)$$

## Stream function wave

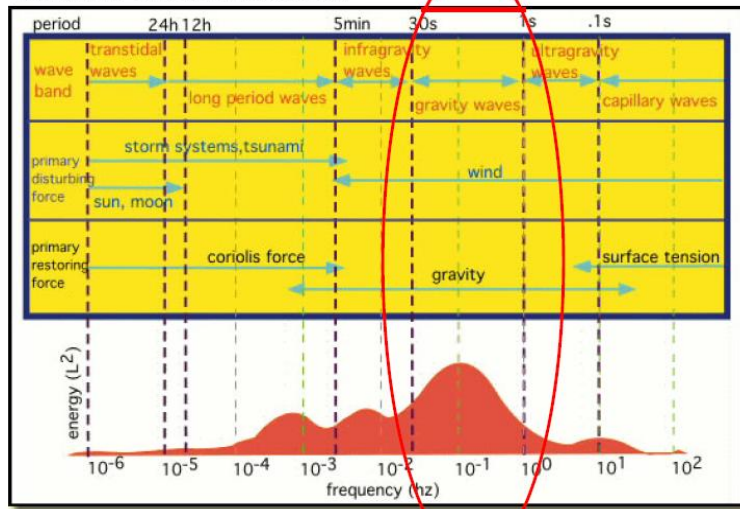
- Stream function waves now added to the wave kinematics generator. (Missing feature for a long time).
- Method by Chaplin, Southampton University
  - Dyn. pressure set to zero so far.

Stream function wave at 44m depth.  $H=8\text{m}$ ,  $T=10\text{s}$



## Irregular Waves(I)

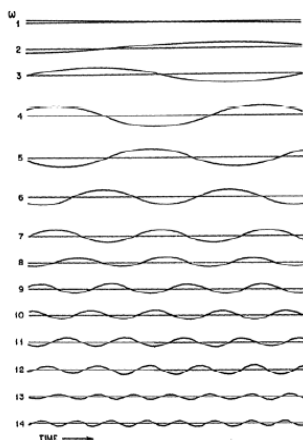
Exemplarily surface wave spectrum



Wind generated

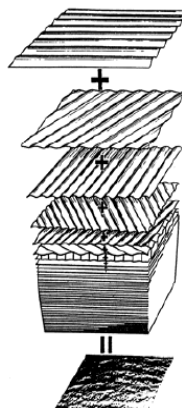
## Irregular wave(III)

Individual linear waves

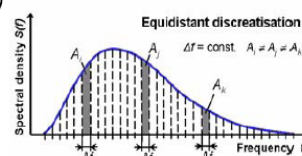


Superposition

(different directions)



Wave spectrum



1. Discretization
2. Single wave amplitudes from spectrum energy
3. Random phases
4. (Directions)
5. Superposition

## Jonswap spectrum for irregular waves:

$$G(\omega) = \frac{\alpha \cdot g^2}{\omega^5} \cdot \exp \left[ -\frac{5}{4} \cdot \left( \frac{\omega_p}{\omega} \right)^4 \right] \gamma^{\exp \left( -\frac{(\omega - \omega_p)^2}{2 \cdot \sigma^2 \cdot \omega_p^2} \right)}$$

$$\alpha = \frac{5}{16} \frac{H_s^2 \omega_p^4}{g^2} (1 - 0.287 \ln(\gamma))$$

$$\omega_p = \frac{2\pi}{T_p}$$

$$\sigma = 0.07 \text{ for } \omega \leq \omega_m, \sigma = 0.09 \text{ for } \omega > \omega_m.$$

## Water kinematics, irregular with spreading directions

$$\alpha_i = \omega_i t - k_i x \cos \theta_i - k_i y \sin \theta_i + \varphi_i \quad \omega_i^2 = g k_i \tanh(k_i z_0)$$

$$\eta_i(t, x, y) = A_i \sin \alpha_i$$

$$\left. \begin{aligned} u_i(t, x, y, z) &= \omega_i \frac{\cosh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \sin \alpha_i \cos \theta_i \\ v_i(t, x, y, z) &= \omega_i \frac{\cosh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \sin \alpha_i \sin \theta_i \\ w_i(t, x, y, z) &= \omega_i \frac{\sinh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \cos \alpha_i \end{aligned} \right\} \text{velocity}$$

$$\eta(t, x, y) = \sum_{i=1}^N \eta_i(t, x, y)$$

$$u(t, x, y, z) = \sum_{i=1}^N u_i(t, x, y, z)$$

$$v(t, x, y, z) = \sum_{i=1}^N v_i(t, x, y, z)$$

$$w(t, x, y, z) = \sum_{i=1}^N w_i(t, x, y, z)$$

$$\left. \begin{aligned} \dot{u}_i(t, x, y, z) &= \omega_i^2 \frac{\cosh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \cos \alpha_i \cos \theta_i \\ \dot{v}_i(t, x, y, z) &= \omega_i^2 \frac{\cosh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \cos \alpha_i \sin \theta_i \\ \dot{w}_i(t, x, y, z) &= -\omega_i^2 \frac{\sinh[k_i(z+z_0)]}{\sinh[k_i z_0]} A_i \sin \alpha_i \end{aligned} \right\} \text{acceleration}$$

$$\dot{u}(t, x, y, z) = \sum_{i=1}^N \dot{u}_i(t, x, y, z)$$

$$\dot{v}(t, x, y, z) = \sum_{i=1}^N \dot{v}_i(t, x, y, z)$$

$$\dot{w}(t, x, y, z) = \sum_{i=1}^N \dot{w}_i(t, x, y, z)$$

$$\left. \begin{aligned} p_i(t, x, y, z) &= \rho g \frac{\cosh[k_i(z+z_0)]}{\cosh[k_i z_0]} A_i \sin \alpha_i \end{aligned} \right\} \text{pressure}$$

$$p(t, x, y, z) = \sum_{i=1}^N p_i(t, x, y, z)$$

## Directional spreading

The directional spreading distribution function

$$f(\theta) = K_{2s} \cos^{2s} \theta \quad K_{2s} = \frac{2^{2s-1} s! (s-1)!}{\pi (2s-1)!}$$

$$s := 2$$

$$K_{2s} := \frac{2^{(2s-1)} s! (s-1)!}{\pi (2s-1)!}$$

The integrated function is calculated

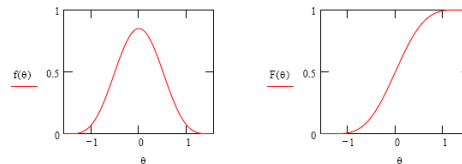
$$F(\theta) = \int_{-\pi/2}^{\theta} f(\theta) d\theta$$

$$f(\theta) := K_{2s} (\cos(\theta))^{2s}$$

$$F(\theta) := \int_{-\pi/2}^{\theta} f(\theta) d\theta$$

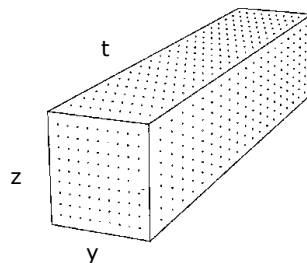
The inverse of this function is calculated so theta can be found with basis of F value.

The uniformly distributed phase angles are used as input to find a corresponding direction angle for each coefficient.



## Wave kinematics

- In the previous version all fourier summations were done in the exact position and time of lookup.
- Now a pregenerated field is created. 3 dimensions when spreading is included. Only height and time resolved for 2D waves.
- 15 points in the height is used. Time resolution is  $T_{min}/10$ .
- Linear interpolation is used.
- A 10 min time series on a jacket that used to take 2hours for simulations, is now ready after 12minutes!

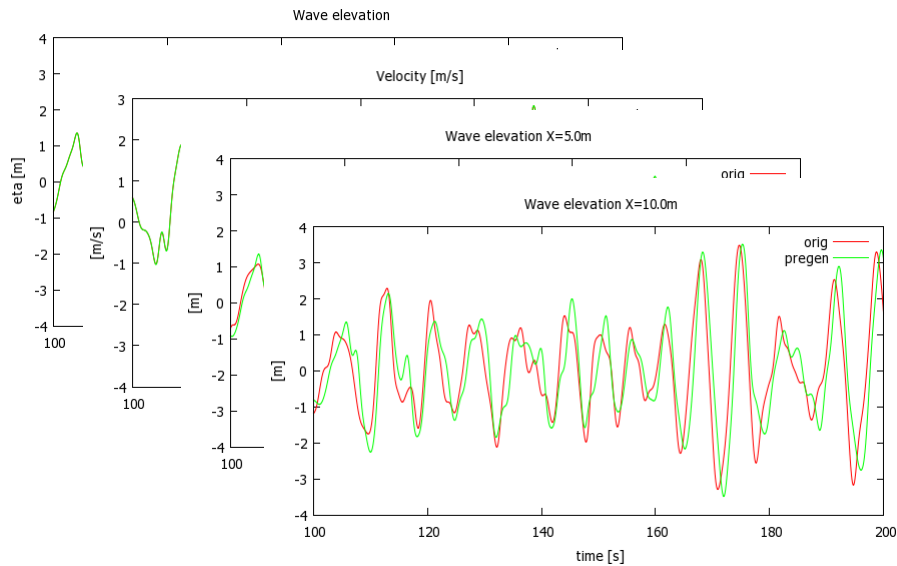


The assumed relation between time and x pos is based on the group velocity of waves.

$$C_g = \frac{1}{2} \sqrt{\frac{g}{k}} \left[ \frac{kh + \tanh(kh) - kh \tanh^2(kh)}{\sqrt{\tanh(kh)}} \right]$$

$$t_{lookup} = t - \frac{x}{C_g}$$

## Comparison



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## Morison formula

For flexible slender structures  $D/L < 0.2$

$$dF = \underbrace{\rho A \dot{U}}_{\text{Froud-Krylov force}} + \underbrace{\rho C_a A_R \dot{U}_{rel}}_{\text{water added mass}} + \underbrace{\frac{1}{2} \rho D C_d U_{rel} |U_{rel}|}_{\text{drag force}}$$

Which for stiff slender structures decouples to

$$dF = \rho \frac{\pi D^2}{4} C_M \dot{U} + \frac{1}{2} \rho D C_d U_{rel} |U_{rel}|, \quad C_M = 1 + C_a$$

- hydrodynamic drag (cd) and inertia (cm=1+ca) coefficients depend on surface roughness, Reynolds number & Keulegan-Carpenter number
- hydrodynamic damping from drag term relevant only for very flexible structures (e.g. Riser, cable)

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# Buoyancy model in HAWC2

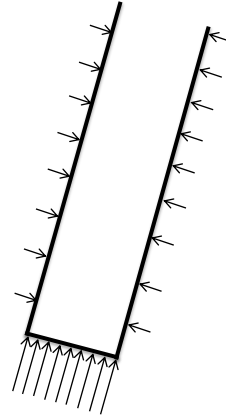
Boyancy implemented as the result of integration of external pressure

- Buoyancy, distributed load contributions:

$$\vec{F}_b = -g\rho \begin{Bmatrix} A_{3,1}S \\ A_{3,2}S \\ -\frac{\partial S}{\partial z}(z-z_0) + \frac{\partial S}{\partial z}p_{dyn} \end{Bmatrix} \quad \vec{M}_b = -g\rho \begin{Bmatrix} -A_{3,2}\frac{\partial r}{\partial z}\pi r^3 \\ A_{3,1}\frac{\partial r}{\partial z}\pi r^3 \\ 0 \end{Bmatrix}$$

- Buoyancy and drag contributions at end nodes

$$F_{b,end}(3) = \underbrace{\rho g S(z-z_0)}_{\text{Static pressure}} + \underbrace{S p_{dyn}}_{\text{Dynamic pressure}} + \underbrace{\frac{1}{2} \rho C_{d,axial} S u_{rel} |u_{rel}|}_{\text{Viscous drag}}$$



A : orientation matrix  
S : area  
r : radius

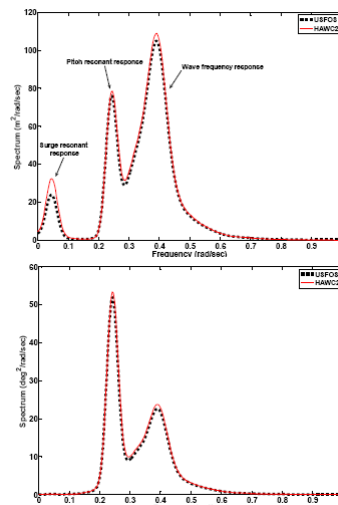
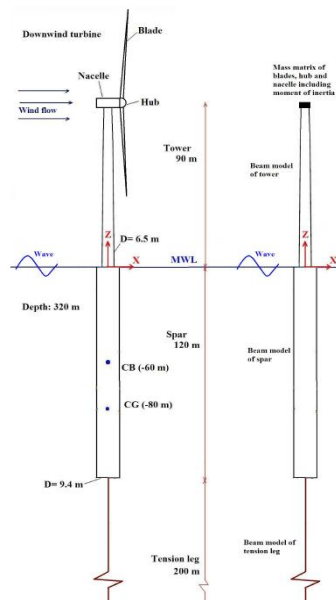
g : gravity  
P : pressure  
ρ : density

z : vertical pos  
u : velocity  
dr/dz : conicity

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# Validation for a spar bouy

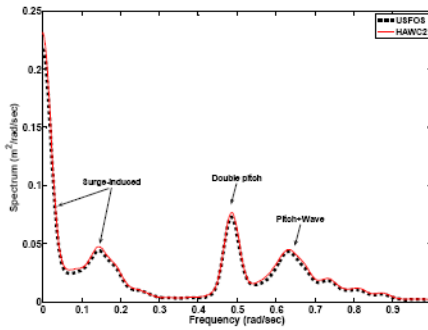


Surge

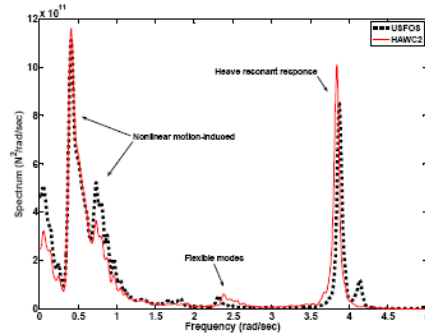
Pitch

Madjid Karimirad, Quentin Meissonnier, Zhen Gao, Torgeir Moan.  
HYDROELASTIC CODE-TO-CODE COMPARISON FOR A TENSION LEG  
SPAR-TYPE FLOATING WIND TURBINE. Submitted to Marine  
Structures jour. 2011.

## Spar bouy (2)



Heave



Tension

## Morison formula Modified for flooded members

$$F = \rho A \ddot{U} + \rho C_a A_R \dot{U}_{rel} + \frac{1}{2} \rho D C_d U_{rel} |U_{rel}|$$

The external forces of a section in balance with acceleration of cylinder and internal flooded water

$$F = m_{cyl} \ddot{u}_{cyl} + \rho A_i \ddot{u}_{cyl}$$

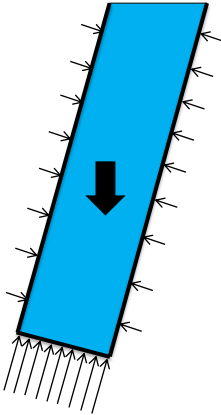
$$\Rightarrow \rho A \ddot{u} + \rho C_m A \dot{u}_{rel} + \frac{1}{2} \rho D C_d u_{rel} |u_{rel}| = m_{cyl} \ddot{u}_{cyl} + \rho A_i \ddot{u}_{cyl}$$

This can be put on same form as the Morisons equation where the effect of the acceleration of flooded water is included in the external force contribution.

$$\Rightarrow \rho(A - A_i) \ddot{u} + \rho(C_m A + A_i) \dot{u}_{rel} + \frac{1}{2} \rho D C_d u_{rel} |u_{rel}| = m_{cyl} \ddot{u}_{cyl}, \quad \dot{u}_{cyl} = \dot{u} - \dot{u}_{rel}$$

## What about flooded members?

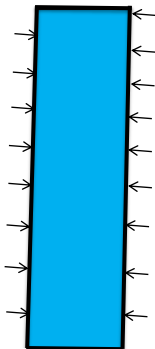
Important for piles in the jacket



- Classical approach in existing aeroelastic codes
  - Internal water applied as extra “steel” mass to ensure correct inertia
- This is not correct since eg. gravity force will accumulate wrongly in the steel structure
- Another method is proposed that is valid for flooded members in compartments without free inner surfaces.

## Flooded members – added mass

We start with the Morisons formula and include the flooded water



$$F_{waves} = m_{steel} \ddot{u}_{cyl} + m_{water} \ddot{u}_{cyl}$$

$$\Rightarrow \rho A \ddot{u} + \rho C_m A_R \ddot{u}_{rel} + \frac{1}{2} \rho D C_d u_{rel} |u_{rel}| = m_{steel} \ddot{u}_{cyl} + \rho A_i \ddot{u}_{cyl}$$

$$\dot{u}_{rel} = \dot{u} - \dot{u}_{cyl}$$

$$\rho(A - A_i) \ddot{u} + \rho(C_m A_R + A_i) \ddot{u}_{rel} + \frac{1}{2} \rho D C_d u_{rel} |u_{rel}| = m_{steel} \ddot{u}_{cyl}$$

And end with a new modified Morisons formula for flooded members

The benefit is an added mass not affected by gravity.

## Buoyancy for flooded members

- The buoyancy is still a result of integration of external pressure
- The flooded water will apply an internal pressure at same location as outer pressure from water
- The result is that the inner area should be subtracted

- Buoyancy, distributed load contributions:

$$\bar{F}_b = -g\rho \begin{Bmatrix} A_{3,1}(S-S_i) \\ A_{3,2}(S-S_i) \\ -\frac{\partial S}{\partial z}(z-z_0) + \frac{\partial S_i}{\partial z}(z-z_0) + \frac{\partial S}{\partial z} p_{dyn} \end{Bmatrix} \quad \bar{M}_b = -g\rho \begin{Bmatrix} -A_{3,2}\pi \left( \frac{\partial r}{\partial z} r^3 - \frac{\partial r_i}{\partial z} r_i^3 \right) \\ A_{3,1}\pi \left( \frac{\partial r}{\partial z} r^3 - \frac{\partial r_i}{\partial z} r_i^3 \right) \\ 0 \end{Bmatrix}$$

- Buoyancy and drag contributions at end nodes

$$F_{b,end}(3) = \rho g (S - S_i)(z - z_0) + S p_{dyn} + \frac{1}{2} \rho C_{d,axial} S u_{rel} |u_{rel}|$$

Static pressure

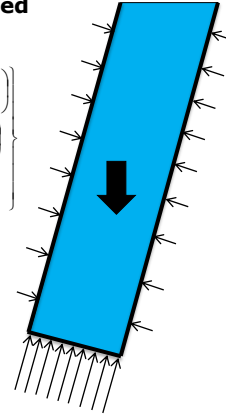
Dynamic pressure

Viscous drag

Also applied at inner nodes when jump in plate thickness occurs

g : gravity  
P : pressure  
ρ : density

Dynamic pressure assumed not to change inner pressure (depends on design though)



## Improvements of solver

A.M. Hansen

- External hydro added mass derived as an analytical integration of the external load.
- Separation of effects from large rotations/movements and local deformation.

Position of a point on a structure

$$\mathbf{u} = \mathbf{R} + \mathbf{A}(\mathbf{r}_c + \mathbf{N}_c \mathbf{q})$$

Acceleration of point

$$\ddot{\mathbf{u}} = \ddot{\mathbf{R}} - \mathbf{A} [\{\mathbf{r}_c\} \times \mathbf{I}] \dot{\bar{\omega}} + \mathbf{A} \mathbf{N}_c \ddot{\mathbf{q}}$$

External force

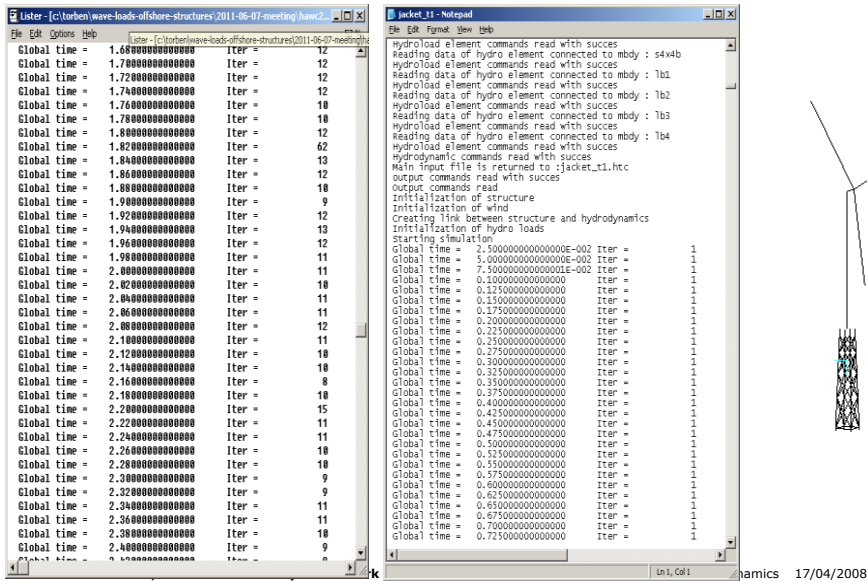
$$\mathbf{Q} = -\mathbf{A} \mathbf{T}_{AS} \mathbf{C}_M \mathbf{T}_{AS}^T \mathbf{A}^T \ddot{\mathbf{u}}$$

$\mathbf{C}_M$  is the section added mass matrix

$$\mathbf{M}_A = \int_L \begin{bmatrix} \mathbf{A} \mathbf{C}_M^S \mathbf{A}^T & \mathbf{A} \mathbf{C}_M^S [\{\mathbf{r}_c\} \times \mathbf{I}] & \mathbf{A} \mathbf{C}_M^S \mathbf{N}_c \\ -[\{\mathbf{r}_c\} \times \mathbf{I}] \mathbf{C}_M^S [\{\mathbf{r}_c\} \times \mathbf{I}] & [\{\mathbf{r}_c\} \times \mathbf{I}] \mathbf{C}_M^S \mathbf{N}_c & \mathbf{N}_c^T \mathbf{C}_M^S \mathbf{N}_c \end{bmatrix} dz$$

where  $\mathbf{C}_M^S = \mathbf{T}_{AS} \mathbf{C}_M \mathbf{T}_{AS}^T$ . Since  $\mathbf{A}$  is part of the added mass matrix and  $\mathbf{A}$  is time dependent, the added mass matrix also becomes time dependent, however,  $\mathbf{A}$  is the *only* time dependent part of the matrix. This means that the added mass matrix have to be updated each time step, but only by pre- and post multiplication by  $\mathbf{A}$  - the remainder of the matrix is integrated only once.

# Better convergence from version 10.4



## HAWC2 example

```
begin HYDRO ;
begin WATER_PROPERTIES ;
rho 1025 ; [kg/m^3]
gravity 9.816 ; [m/s^2]
mw1 0.000; [m]
mudlevel 14.500 ; [m]
water_kinematics_dll ./wkin_dll.dll ./hydrohtc/reg_airy_1.htc ;
end WATER_PROPERTIES ;
;
begin HYDRO_ELEMENT ;
mbdy_name L1 ;
buoyancy 1 ;
update_states 1 ; (0: no dynamic interaction, 1: fully coupled solution
hydrosections auto 4 ; dist. of hydro calculation points from 1 to nsec
nsec 9 ; z Cm Cd A Aref width dr/dz Cd_a(quad) Cm_a Cd_a_lin Ai
sec 0.000 1 1 1.1309734 1.1309734 1.20 0.0 0.0 0.0 0.0 0.9503318 ;
sec 5.005 1 1 1.1309734 1.1309734 1.20 0.0 0.0 0.0 0.0 0.9503318 ;
sec 5.015 1 1 1.5393804 1.5393804 1.40 0.0 0.0 0.0 0.0 0.9503318 ;
sec 20.408 1 1 1.5393804 1.5393804 1.40 0.0 0.0 0.0 0.0 0.9503318 ;
sec 20.418 1 1 1.5393804 1.5393804 1.40 0.0 0.0 0.0 0.0 1.0028749 ;
sec 43.046 1 1 1.5393804 1.5393804 1.40 0.0 0.0 0.0 0.0 1.0028749 ;
sec 43.056 1 1 1.1309734 1.1309734 1.20 0.0 0.0 0.0 0.0 1.0028749 ;
sec 49.431 1 1 1.1309734 1.1309734 1.20 0.0 0.0 0.0 0.0 1.0028749 ;
sec 61.215 1 1 1.1309734 1.1309734 1.20 0.0 0.0 0.0 0.0 1.0028749 ;
end HYDRO_ELEMENT ;
end HYDRO ;
```

## Wkin\_dll input file example, regular airy

```
begin wkin_input ;
  wavetype 0 ; 0=regular, 1=irregular, 2=deterministic
  wdepth 220.0 ;
;
  begin reg_airy ;
    stretching 0; 0=none, 1=wheeler
    wave 9 12.6; Hs,T
  end;
end;
;
exit ;
```

## Wkin\_dll input file example, irregular airy

```
begin wkin_input ;
  wavetype 1 ; 0=regular, 1=irregular, 2=deterministic
  wdepth 220.0 ;
;
  begin irreg_airy ;
    stretching 0; 0=none, 1=wheeler
    spectrum 1; (1=jonswap)
    jonswap 9 12.6 3.3 ; (Hs, Tp, gamma)
    coef 200 1 ; (coefnr, seed)
    spreading 1 2; (type(0=off 1=on), s parameter (pos. integer min 1)
  end;
end;
;
exit ;
```

## Wkin\_dll input file example, deterministic airy

```
begin wkin_input ;
  wavetype 2 ; 0=regular, 1=irregular, 2=deterministic
  wdepth 220.0 ;
;
begin det_airy ;
  stretching 0; 0=none, 1=wheeler
  file ../waves/elevation.dat ;
  nsamples 32768 ;
  nskip 1 ;
  columns 1 2 ; time column, elevation column
end;
end;
exit ;
```

### ../waves/elevation.dat file example

```
time, elevation
0.0 0.0
0.1 0.2
0.2 0.4
0.3 0.6
0.4 0.8
0.5 0.9
0.6 1.0
0.7 0.9
0.8 0.8
```

## Wkin\_dll input file example, stream function

```
begin wkin_input ;
  wavetype 3 ; 0=regular, 1=irregular, 2=deterministic, 3=stream function
  wdepth 40.0 ;
;
begin strf ;
  wave 8.0 10.0 0.0 ; Hs,T,current
end;
end;
;
exit ;
```